Tampere University

Mathematical Systems Theory Research Group

Computing Sciences, ITC Faculty

Mathematical Systems Theory

Mathematical systems theory studies the behaviour and properties of dynamical models that are described by ordinary and partial differential equations, as well as other models with dynamical behaviour. Such **dynamical systems** arise from modelling phenomena that are encountered in engineering applications and in the nature.

Mathematical control theory concentrates on influencing the behaviour of such dynamical models to either improve the properties of the system or to drive the system into a desirable trajectory.

Mathematical systems theory is used in, for example,

- Trajectory control for unmanned vehicles, such as quadrocopters
- Automatic control of self-driving cars
- Rejection of disturbances in industrial processes and acoustics
- Removal of unwanted vibrations in buildings and in machinery

Linear Control Systems

In linear systems theory the considered models are of the form

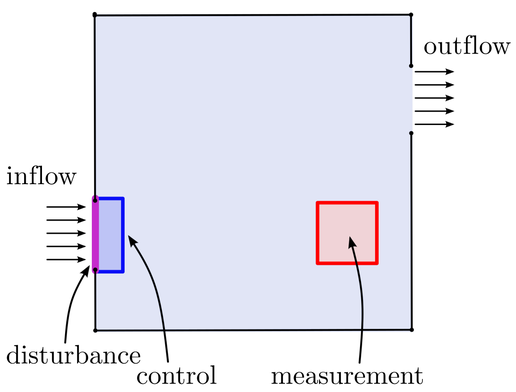
$$\frac{d}{dt}x(t) = Ax(t) + Bu(t), \quad x(0) = x_0 \tag{1a}$$

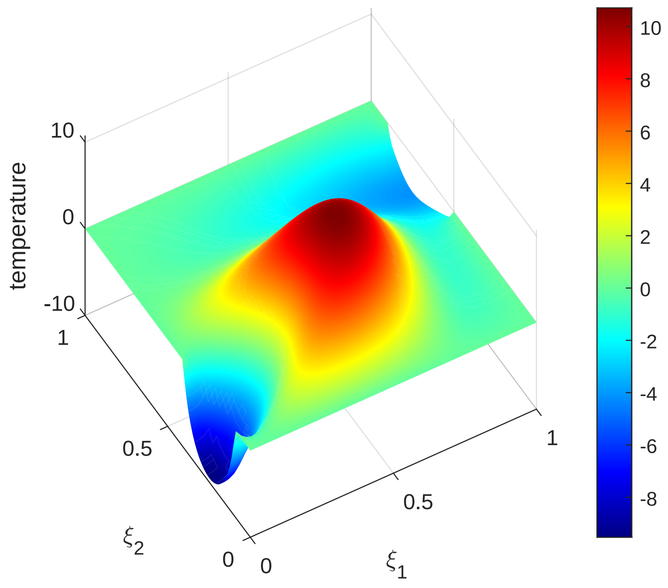
$$y(t) = Cx(t) + Du(t). \tag{1b}$$

Here $x(t)$ is the **state** of the system, $y(t)$ is the measured **output**, $u(t)$ the **control input**. The Systems Theory Research Group at TAU focuses on the control of models involving **partial differential equations** and **infinite-dimensional systems**. This class of models facilitates the control of processes involving, in particular,

- Heat diffusion and fluid flows
- Vibrations and wave phenomena
- Elastic deformations and flexible structures
- Flows and transport on networks

The following figures illustrate the control of a room temperature profile (modelled with linearized "**Boussinesq equations**").





Research Topics

If a process is modelled with ordinary differential equations, A, B, C and D are matrices. In the case of linear partial differential equations, (1a) becomes an abstract differential equation, and A, B, C and D are **linear operators**. Such systems can be studied using linear **functional analysis**, and especially **operator theory**.

Typical research questions in systems theory include the following:

- Solvability of a given control problem
- Characterization and construction of controllers
- Optimality of a given control design
- Study of long-time behaviour of solutions $x(t)$ of (1a)
- Numerical approximations and implementation of controllers

Selected publications

R. Chill, L. Paunonen, D. Seifert, R. Stahn, and Y. Tomilov. Non-uniform stability of damped contraction semigroups. *Analysis & PDE*, 2022.

K. Huhtala, L. Paunonen, and Weiwei Hu. Robust output regulation of the linearized Boussinesq equations with boundary control and observation. *Mathematics of Control, Signals, and Systems*, 2022.

L. Paunonen and D. Phan. Reduced order controller design for robust output regulation. *IEEE Transactions on Automatic Control*, 2020.

J.-P. Humaloja, M. Kurula and L. Paunonen. Approximate robust output regulation of boundary control systems. *IEEE Transactions on Automatic Control*, 2019.

L. Paunonen. Controller design for robust output regulation of regular linear systems. *IEEE Transactions on Automatic Control*, 2016.

Courses at Tampere University

"Introduction to Functional Analysis" A first course on functional analysis, including fundamentals of the theory of function spaces, operator theory, and spectral theory.

"Mathematical Control Theory" A second course on functional analysis with emphasis on applications to partial differential equations, infinite-dimensional dynamical systems, and control theory.

Research group Members

Lassi Paunonen – Associate professor, group leader
Nicolas Vanspranghe – Post-doctoral researcher
Thavamani Govindaraj – Post-graduate student
Mikko Lehtimäki – Post-graduate student
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