The Internal Model Principle for Regular Linear Systems

Lassi Paunonen

Tampere University of Technology, Finland

May 5th, 2015
TUT Workshop on Control
Tampere

Main Objectives

Problem

Introduce new results on robust output regulation for regular linear systems.

Main results:

- Characterization of controllers achieving output regulation via the *regulator equations*
- Characterization of robust controllers via the Internal Model Principle. In addition:
    - A test to determine robustness with respect to a given set of perturbations.
    - Theory of Reduced Order Internal Models
- Robust controllers for regular linear systems

The Infinite-Dimensional Plant

Consider a plant

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \qquad x(0) = x_0 \in X \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

where

- $u(t) \in U = \mathbb{C}^m$ is the control input
- $y(t) \in Y = \mathbb{C}^p$ is the measured output.

The Infinite-Dimensional Plant

Consider a plant

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t), \qquad x(0) = x_0 \in X\\ y(t) &= Cx(t) + Du(t)\end{aligned}$$

More general assumptions on B and C:

- $B \in \mathcal{L}(\mathbb{C}^m, X_{-1})$ and $C \in \mathcal{L}(X_1, \mathbb{C}^p)$ are admissible
- Define $C_\Lambda x = \lim_{\lambda\to\infty} \lambda CR(\lambda, A)x$ in $\mathcal{D}(C_\Lambda) = \{\, x \mid \text{lim exists} \,\}$
- (A, B, C, D) is regular, i.e., for one/all $\lambda \in \rho(A)$

$$P(\lambda) = C_\Lambda R(\lambda, A_{-1})B + D$$

is well-defined and $P(\cdot) \in H^\infty(\mathbb{C}_\beta^+)$ for some $\beta \in \mathbb{R}$.

The Infinite-Dimensional Plant

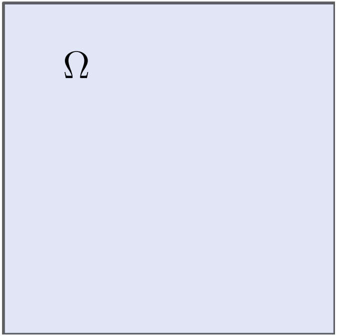
Consider a plant

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t), \qquad x(0) = x_0 \in X\\ y(t) &= Cx(t) + Du(t)\end{aligned}$$

Properties: For any $\beta > \omega_0(T(t))$ we have

$$\begin{aligned}P(\cdot) &\in H^\infty(\mathbb{C}_\beta^+, \mathcal{L}(U,Y)),\\ C_\Lambda R(\cdot, A) &\in H^\infty(\mathbb{C}_\beta^+, \mathcal{L}(X,Y)),\\ R(\cdot, A_{-1})B &\in H^\infty(\mathbb{C}_\beta^+, \mathcal{L}(U,X)),\end{aligned}$$

Example: Controlled Heat Equation



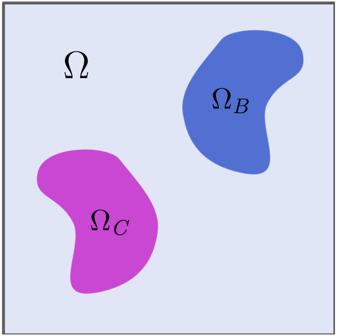
A 2D heat equation on Ω with Dirichlet BC's:

$$u_t = \Delta u, \quad u(\xi,0) = u_0(\xi)$$
$$u(t,\xi)|_{\partial\Omega} = 0.$$

The system on $X = L^2(\Omega)$ with

$$A = \Delta : \mathcal{D}(A) \subset X \to X$$
$$\mathcal{D}(A) = H^2(\Omega) \cap H_0^1(\Omega)$$

Example: Controlled Heat Equation



The system on $X = L^2(\Omega)$ with

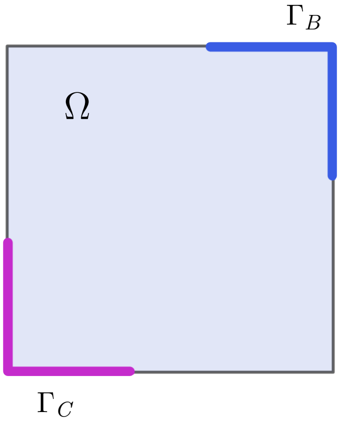
$$A = \Delta : \mathcal{D}(A) \subset X \to X$$
$$\mathcal{D}(A) = H^2(\Omega) \cap H_0^1(\Omega)$$

Distributed control and obs.:

$$Cx(t) = \int_{\Omega_C} x(\xi, t) d\xi \in \mathbb{C}$$
$$Bu(t) = \chi_{\Omega_B}(\cdot) u(t), \quad u(t) \in \mathbb{C}$$

Here B and C are bounded.

Example: Heat Equation with Boundary Control



The system on $X = L^2(\Omega)$ with

$$A = \Delta : \mathcal{D}(A) \subset X \to X$$
$$\mathcal{D}(A) = H^2(\Omega) \cap H_0^1(\Omega)$$

Control and observation also possible through the boundary (Dirichlet/Neumann/Robin)

Here B and C are unbounded, and the plant is regular under suitable assumptions.

The Control Problem

Problem (Robust Output Regulation)

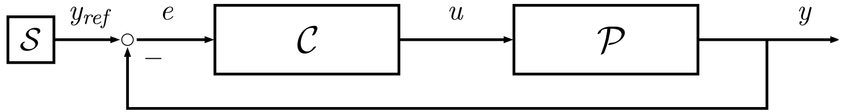
Choose a control law in such a way that

- *The output $y(t)$ tracks a given reference signal $y_{ref}(t)$ asymptotically, i.e.*

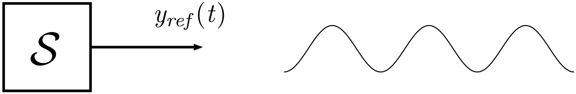
$$\lim_{t\to\infty} \|y(t) - y_{ref}(t)\| = 0$$

- *The above property is robust with respect to small perturbations in the operators (A, B, C, D) of the plant.*

The Exosystem and the Control Scheme



The Exosystem

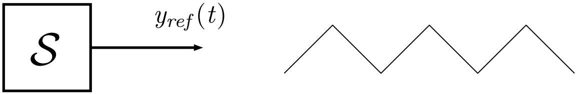


The reference signals can be trigonometric polynomials

$$y_{\mathit{ref}}(t) = \sum_{k=1}^{q} c_k e^{i\omega_k t}$$

The signal $y_{ref}(t)$ contains the frequencies $(i\omega_k)_{k=1}^{q} \in i\mathbb{R}$.

The Exosystem

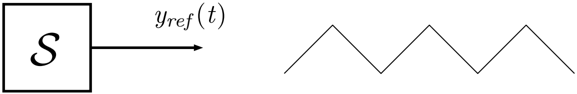


Alternatively, we can consider nonsmooth periodic or almost periodic functions

$$y_{ref}(t) = \sum_{k\in\mathbb{Z}} c_k e^{i\omega_k t}$$

The signal $y_{ref}(t)$ contains the frequencies $(i\omega_k)_{k\in\mathbb{Z}} \in i\mathbb{R}$.

The Exosystem



The references y_{ref} are obtained as outputs of the *exosystem*

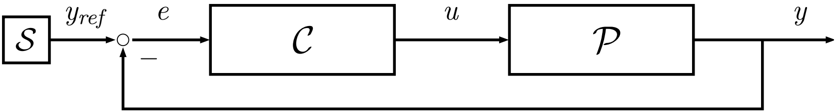
$$\begin{aligned} \dot{v}(t) &= Sv(t), \qquad v_0 \in W \\ y_{ref}(t) &= Fv(t) \end{aligned}$$

where S is an unbounded diagonal operator

$$S = \mathrm{diag}(i\omega_k)_{k\in\mathbb{Z}}, \qquad \mathcal{D}(S) = \{\, (v_k) \in W \mid (\omega_k v_k) \in \ell^2(\mathbb{C}) \,\}$$

on the Hilbert space $W = \ell^2(\mathbb{C})$ and $F \in \mathcal{L}(W, Y)$. The eigenvalues $i\omega_k \in i\mathbb{R}$ of S are the frequencies in $y_{ref}(t)$.

The Exosystem and the Control Scheme



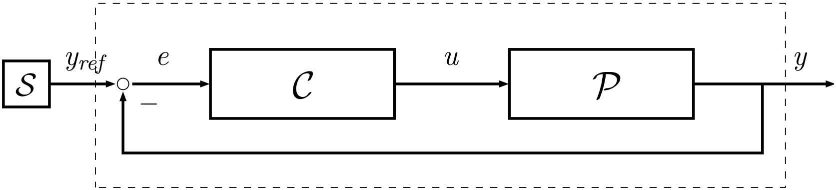
The Dynamic Error Feedback Controller

We consider an error feedback controller $(\mathcal{G}_1, \mathcal{G}_2, K)$ of the form

$$\begin{aligned} \dot{z}(t) &= \mathcal{G}_1 z(t) + \mathcal{G}_2(y(t) - y_{ref}(t)), \qquad z(0) = z_0 \in Z \\ u(t) &= Kz(t), \end{aligned}$$

where $\mathcal{G}_1$ generates a semigroup on a Banach space Z, $\mathcal{G}_2 \in \mathcal{L}(Y, Z)$ and $K \in \mathcal{L}(Z_1, U)$ is admissible.

The Closed-Loop System



Closed-loop system

Theorem

The closed-loop system (A_e, B_e, C_e, D_e) *consisting of the plant and the controller is a regular linear system.*

Proof.

Theory of feedback for regular linear systems, by Weiss (1994). □

The Control Problem

The Robust Output Regulation Problem on W_α

Choose a controller $(\mathcal{G}_1, \mathcal{G}_2, K)$ in such a way that

- The closed-loop system is strongly/polynomially/exponentially stable.
- The output $y(t)$ tracks a given reference signal $y_{ref}(t)$ asymptotically, i.e.

$$\lim_{t\to\infty} \|y(t) - y_{ref}(t)\| = 0$$

  for all initial states $x_{e0} = (x_0, z_0) \in \mathcal{D}(A_e)$ and $v_0 \in W_\alpha$
- The above property is robust with respect to small perturbations in the operators (A, B, C, D) of the plant.

The Internal Model Principle

Theorem (Francis & Wonham, 1970's, LP & SP 2010)

A stabilizing feedback controller solves the robust output regulation problem if and only if it contains p copies of the dynamics of the signal generator.

Here $p = \dim Y$, the number of outputs.

The Internal Model Principle

Theorem (Francis & Wonham, 1970's, LP & SP 2010)

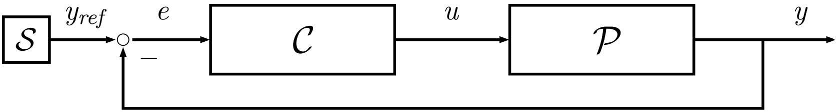
A stabilizing feedback controller solves the robust output regulation problem if and only if it contains p copies of the dynamics of the signal generator.

Here $p = \dim Y$, the number of outputs.

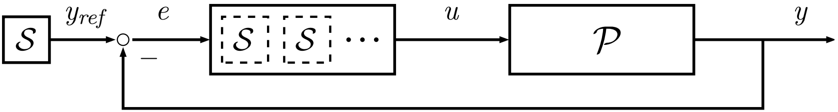
The p-copy for an exosystem with $S = \mathrm{diag}(i\omega_k)_{k\in\mathbb{Z}}$:

> *Any eigenvalue $i\omega_k$ of S must be an eigenvalue of $\mathcal{G}_1$ with p linearly independent eigenvectors associated to it, i.e.,*
>
> $$\dim \mathcal{N}(i\omega_k - \mathcal{G}_1) \geq \dim Y = p.$$

Feedback Controller



The p-Copy Internal Model Principle



Remarks on the Internal Model Principle

Remark

The proof of the Internal Model Principle is largely based on requiring robustness with respect to perturbations to the output operators of the exosystem.

Allowing such perturbations is often unnecessary (in particular, if reference signals are known accurately).

Motivates the study of robustness with respect to "smaller" classes of perturbations, and for individual perturbations.

A Bit More Assumptions...

To make things simpler, assume the following:

- Equal number of inputs and outputs, $P(\lambda) \in \mathbb{C}^{p \times p}$.
- $P(i\omega_k)$ invertible for all k (a standard assumption).

Basic Assumptions on the Perturbations

Denote by $\mathcal{O}$ the class of all admissible perturbations of the plant:

$$(A, B, C, D) \quad \longrightarrow \quad (\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) \in \mathcal{O}.$$

Basic Assumptions on the Perturbations

Denote by $\mathcal{O}$ the class of all admissible perturbations of the plant:

$$(A, B, C, D) \quad \longrightarrow \quad (\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) \in \mathcal{O}.$$

The perturbations in $\mathcal{O}$ are assumed be "small" so that

- The perturbed closed-loop system is exponentially stable
- $\tilde{P}(i\omega_k)$ exists and is invertible for all $k \in \{1, \ldots, q\}$.

Here

$$\tilde{P}(\lambda) = \tilde{C}R(\lambda, \tilde{A})\tilde{B} + \tilde{D}.$$

Aim

Problem (Robust Output Regulation)

The controller $(\mathcal{G}_1, \mathcal{G}_2, K)$ *is such that*

- *The output* $y(t)$ *tracks the reference signal* $y_{ref}(t)$*, i.e.*

$$\lim_{t \to \infty} \|y(t) - y_{ref}(t)\| = 0 \tag{1}$$

- *If the operators of the plant are changed s.t.*

$$(A, B, C, D) \quad \longrightarrow \quad (\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) \in \mathcal{O},$$

the property (1) *is still true.*

If the second part is true for some $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$, we say that the controller is *robust* w.r.t. to these perturbations.

Testing Robustness for Perturbations in $\mathcal{O}$

Theorem (LP 2013)

A stabilizing controller $(\mathcal{G}_1, \mathcal{G}_2, K)$ is robust with respect to given perturbations $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) \in \mathcal{O}$ if and only if the equations

$$\tilde{P}(i\omega_k) K z^k = -F e_k$$

$$(i\omega_k - \mathcal{G}_1) z^k = 0$$

have a solution $z^k \in \mathcal{D}(\mathcal{G}_1)$ for all $k \in \{1, \dots, q\}$.

Here: e_k is an Euclidean basis vector, F is the output operator of the exosystem, $\mathcal{G}_1$ is the system operator and K the output operator of the controller.

Example: A MIMO Wave Equation

Set-point regulation ($y_{ref}(t) \equiv y_r \in \mathbb{C}^p$ constant, $p > 1$) for

$$\frac{\partial^2 w}{\partial t^2}(z,t) - \alpha \frac{\partial w}{\partial t}(z,t) = \frac{\partial^2 w}{\partial z^2}(z,t) + Bu(t)$$

$$y(t) = Cw(\cdot,t).$$

Require robustness w.r.t changes in α.

Example: A MIMO Wave Equation

Set-point regulation ($y_{ref}(t) \equiv y_r \in \mathbb{C}^p$ constant, $p > 1$) for

$$\frac{\partial^2 w}{\partial t^2}(z,t) - \alpha \frac{\partial w}{\partial t}(z,t) = \frac{\partial^2 w}{\partial z^2}(z,t) + Bu(t)$$

$$y(t) = Cw(\cdot, t).$$

Require robustness w.r.t changes in α.

Example

We can build a 1-dimensional controller that is robust with respect to all sufficiently small perturbations in α.

Example: A MIMO Wave Equation

Set-point regulation ($y_{ref}(t) \equiv y_r \in \mathbb{C}^p$ constant, $p > 1$) for

$$\frac{\partial^2 w}{\partial t^2}(z,t) - \alpha \frac{\partial w}{\partial t}(z,t) = \frac{\partial^2 w}{\partial z^2}(z,t) + Bu(t)$$

$$y(t) = Cw(\cdot, t).$$

Require robustness w.r.t changes in α.

Example

We can build a 1-dimensional controller that is robust with respect to all sufficiently small perturbations in α.

Key: Exosystem has $i\omega_0 = 0$, and for perturbations in α we have $\tilde{P}(0) = P(0)$. Thus one copy of the exosystem is sufficient.

Controller Construction

Problem

For a given class $\mathcal{O}_0$ of admissible perturbations, design an error feedback controller of the form

$$\dot{z}(t) = \mathcal{G}_1 z(t) + \mathcal{G}_2(y(t) - y_{ref}(t)), \qquad z(0) = z_0 \in Z$$
$$u(t) = Kz(t),$$

such that the controller is robust with respect to the class $\mathcal{O}_0$ of perturbations.

In particular, we require robustness w.r.t those perturbations in $\mathcal{O}_0$ *for which the closed-loop stability is preserved*.

Main Result

Theorem (LP 2015)

For every $k \in \{1, \dots, q\}$ *define*

$$\mathcal{S}_k = \operatorname{span}\{\, \tilde{P}(i\omega_k)^{-1} F \phi_k \mid (\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) \in \mathcal{O}_0 \,\} \subset \mathbb{C}^p$$

and $p_k = \dim \mathcal{S}_k$.

The robust output regulation problem can be solved with a reduced order internal model G_1 *satisfying*

$$\dim \mathcal{N}(i\omega_k - G_1) = p_k \qquad k \in \{1, \dots, q\}.$$

The Structure of the Controller

We use a triangular structure,

$$\dot{z}(t) = \begin{pmatrix} G_1 & R_1 \\ 0 & R_2 \end{pmatrix} z(t) + \begin{pmatrix} G_2 \\ R_3 \end{pmatrix} (y(t) - y_{ref}(t))$$

$$u(t) = \begin{pmatrix} K_1 & K_2 \end{pmatrix} z(t)$$

- (G_1, K_1) is the reduced order internal model of the exosystem
- Other operators are used in stabilization ("observer"-based)

The Structure of the Controller

We use a triangular structure,

$$\dot{z}(t) = \begin{pmatrix} G_1 & R_1 \\ 0 & R_2 \end{pmatrix} z(t) + \begin{pmatrix} G_2 \\ R_3 \end{pmatrix} (y(t) - y_{ref}(t))$$
$$u(t) = \begin{pmatrix} K_1 & K_2 \end{pmatrix} z(t)$$

Here

$$G_1 = \begin{pmatrix} i\omega_1 I_{p_1} & & \\ & \ddots & \\ & & i\omega_q I_{p_q} \end{pmatrix},$$

where $i\omega_k I_{p_k} \in \mathbb{C}^{p_k \times p_k}$, and K_1 is defined in terms of a basis of

$$\mathcal{S}_k = \operatorname{span}\{ \tilde{P}(i\omega_k)^{-1} F \phi_k \mid (\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) \in \mathcal{O}_0 \} \subset \mathbb{C}^p$$

Example: Coupled Harmonic Oscillators

Consider two interconnected driven harmonic oscillators

$$\begin{aligned} \ddot{q}_1(t) + \alpha_1 \dot{q}_1(t) + q_1(t) &= q_2(t) + F_1(t) \\ \ddot{q}_2(t) + \alpha_2 \dot{q}_2(t) + 2q_2(t) &= -q_1(t) + F_2(t), \end{aligned}$$

where the damping coefficients $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$ are subject to perturbations. Initially $\alpha_1 = 1$ and $\alpha_2 = 0$.

Example: Coupled Harmonic Oscillators

Consider two interconnected driven harmonic oscillators

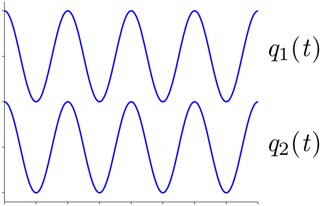
$$\begin{aligned} \ddot{q}_1(t) + \alpha_1 \dot{q}_1(t) + q_1(t) &= q_2(t) + F_1(t) \\ \ddot{q}_2(t) + \alpha_2 \dot{q}_2(t) + 2q_2(t) &= -q_1(t) + F_2(t), \end{aligned}$$

We control the forces $F_1(t)$ and $F_2(t)$ and aim at

$$\left\| \begin{pmatrix} q_1(t) \\ q_2(t) \end{pmatrix} - y_{ref}(t) \right\| \longrightarrow 0,$$

for

$$y_{ref}(t) = \begin{bmatrix} 1 + \cos(\pi t) \\ -1 + \cos(\pi t) \end{bmatrix}$$



The Robust Controller

The reference signal contains three frequencies

$$i\omega_0 = 0, \qquad i\omega_{\pm 1} = \pm i\pi$$

The perturbations in the damping coefficients only affect A, and analyzing

$$\mathcal{S}_k = \operatorname{span}\{\, \tilde{P}(i\omega_k)^{-1} F \phi_k \mid (\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) \in \mathcal{O}_0 \,\} \subset \mathbb{C}^2$$

we have

$$\begin{cases} p_0 & = \dim \mathcal{S}_0 = 1 \\ p_{\pm 1} & = \dim \mathcal{S}_{\pm 1} = 2 \end{cases}$$

due to the observation that $\tilde{P}(0) = P(0)$.

The Reduced Order Internal Model

Due to

$$\begin{cases} p_0 & = \dim \mathcal{S}_0 = 1 \\ p_{\pm 1} & = \dim \mathcal{S}_{\pm 1} = 2 \end{cases}$$

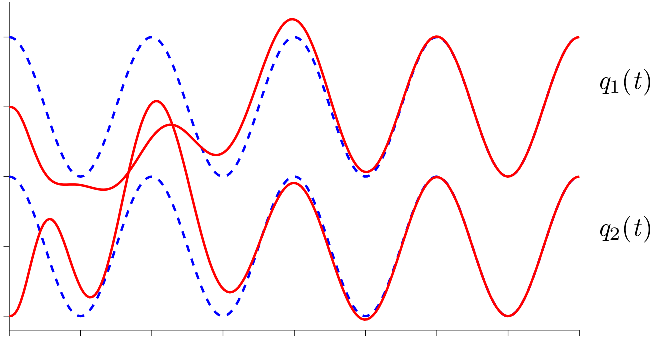
we can design a controller with a reduced order internal model

$$G_1 = \begin{pmatrix} i\omega_{-1} & & & & \\ & i\omega_{-1} & & & \\ & & i\omega_0 & & \\ & & & i\omega_1 & \\ & & & & i\omega_1 \end{pmatrix} = \begin{pmatrix} -i\pi & & & & \\ & -i\pi & & & \\ & & 0 & & \\ & & & i\pi & \\ & & & & i\pi \end{pmatrix}$$

such that small changes in the damping coefficients do not destroy output tracking.

Simulation

Behaviour of the systems with the controller:



The Controller Structure

Our controller has a structure

$$\begin{aligned}\dot{z}(t) &= \begin{pmatrix} G_1 & R_1 \\ 0 & R_2 \end{pmatrix} z(t) + \begin{pmatrix} G_2 \\ R_3 \end{pmatrix} (y(t) - y_{ref}(t)) \\ u(t) &= \begin{pmatrix} K_1 & K_2 \end{pmatrix} z(t)\end{aligned}$$

- (G_1, K_1) is the reduced order internal model of the exosystem
- Other operators are used in stabilization

Comparison: Structure by Hämäläinen & Pohjolainen'10

The triangular structure (based on Francis & Wonham '74)

$$\dot{z}(t) = \begin{pmatrix} G_1 & 0 \\ R_1 & R_2 \end{pmatrix} z(t) + \begin{pmatrix} G_2 \\ R_3 \end{pmatrix} (y(t) - y_{ref}(t))$$

$$u(t) = \begin{pmatrix} K_1 & K_2 \end{pmatrix} z(t)$$

- (G_1, G_2) contain the internal model of the exosystem
- Other operators are used in stabilization (observer)

Pros and cons:

(+) Natural if the IM is defined via *$\mathcal{G}$-conditions* (F&W'74).

(−) Difficult for the p-copy version of the internal model

(−) Does not work for reduced order internal models.

The Controller Structure

Our controller has a structure

$$\dot{z}(t) = \begin{pmatrix} G_1 & R_1 \\ 0 & R_2 \end{pmatrix} z(t) + \begin{pmatrix} G_2 \\ R_3 \end{pmatrix} (y(t) - y_{ref}(t))$$
$$u(t) = \begin{pmatrix} K_1 & K_2 \end{pmatrix} z(t)$$

- (G_1, K_1) is the reduced order internal model of the exosystem
- Other operators are used in stabilization

Pros and cons:

$(+)$ Natural if the internal model is defined using the p-copy

$(+)$ Can be used for reduced order internal models!

$(-)$ Difficult for $\mathcal{G}$-conditions

Comparison of the Two Structures

The Old One: $K = (K_1,\ K_2)$

$$\mathcal{G}_1 = \begin{pmatrix} G_1 & 0 \\ R_1 & R_2 \end{pmatrix}, \quad \mathcal{G}_2 = \begin{pmatrix} G_2 \\ R_3 \end{pmatrix}$$

- (G_1, G_2) is the IM
- $(R_1, R_2, R_3, K_1, K_2)$ used in stabilization

(+) Good for $\mathcal{G}$-conds

The New One: $K = (K_1,\ K_2)$

$$\mathcal{G}_1 = \begin{pmatrix} G_1 & R_1 \\ 0 & R_2 \end{pmatrix}, \quad \mathcal{G}_2 = \begin{pmatrix} G_2 \\ R_3 \end{pmatrix}$$

- (K_1, G_1) is the IM
- $(R_1, R_2, R_3, G_2, K_2)$ used in stabilization

(+) Good for p-copy and ROIM

Comparison: Choices of R_1, R_2, R_3

The Old One:

$$\mathcal{G}_1 = \begin{pmatrix} G_1 & 0 \\ (B+LD)K_1 & A_{-1} + BK_2 + L(C_\Lambda + DK_2) \end{pmatrix}, \quad \mathcal{G}_2 = \begin{pmatrix} G_2 \\ -L \end{pmatrix},$$

The New One:

$$\mathcal{G}_1 = \begin{pmatrix} G_1 & G_2(C + DK_2) \\ 0 & A_{-1} + BK_2 + L(C_\Lambda + DK_2) \end{pmatrix}, \quad \mathcal{G}_2 = \begin{pmatrix} G_2 \\ L \end{pmatrix},$$

Where $(A_{-1} + BK_2)|_X$ and $A + LC_\Lambda$ are exponentially stable.

Theorem

The controllers with natural domains $\mathcal{D}(\mathcal{G}_1)$ generate C_0-sg's, and solve the RORP in such a way that the CL system is strongly stable

$$\|R(i\omega_k, A_e)\| = M(1 + \|P(i\omega_k)^{-1}\|^2)$$

Polynomial Stability and The Class of Reference Signals

Theorem

The controllers solve the RORP and

$$\|R(i\omega_k, A_e)\| = M(1 + \|P_K(i\omega_k)^{-1}\|^2 \|K_{1k}\|^2)$$

The bound can also be improved to nonuniform stability of the closed-loop system: If $\alpha > 0$ is such that

$$\|P_K(i\omega_k)^{-1}\| \lesssim |\omega_k|^{-\alpha}$$

we can stabilize in such a way that

$$\|R(i\omega, A_e)\| \leq M(1 + |\omega_k|^{2\alpha+1+\varepsilon})$$

and

$$\|T_e(t)x_{e0}\| \leq \frac{M_e}{t^{1/(2\alpha+1+\varepsilon)}} \|A_e x_{e0}\|, \qquad \forall x_{e0} \in \mathcal{D}(A_e)$$

Polynomial Stability and The Class of Reference Signals

Theorem

The controllers solve the RORP and

$$\|R(i\omega_k, A_e)\| = M(1 + \|P_K(i\omega_k)^{-1}\|^2 \|K_{1k}\|^2)$$

The class of regulated signals correspond to initial states $v_0 \in W$ of the exosystem for for which

$$\Big((1 + \|P_K(i\omega_k)^{-1}\|^2 \|K_{1k}\|^2)\langle v_0, \phi_k\rangle\Big)_{k\in\mathbb{Z}} \in \ell^2(\mathbb{C})$$

If $\dim Y < \infty$, this is satisfied if

$$\Big((1 + \|P(i\omega_k)^{-1}\|^2 \|K_{1k}\|^2)\langle v_0, \phi_k\rangle\Big)_{k\in\mathbb{Z}} \in \ell^2(\mathbb{C})$$

References

LP & S. Pohjolainen - *Reduced order internal models in robust output regulation*, Transactions on Automatic Control, 2013.

LP - *Designing controllers with reduced order internal models*, Transactions on Automatic Control, 2015.

LP & S. Pohjolainen - *The internal model principle for systems with unbounded control and observation*, SIAM Journal on Control & Optimization, 2015.

Extensions

The results in this presentation are also valid for

- Also for disturbance rejection
- Non-diagonal exosystems (i.e., S has Jordan blocks)
- Infinite-dimensional exosystems (nonsmooth reference signals)
- Infinite-dimensional plants with unbounded control and observation.

Conclusions

In this presentation.

- Robust output regulation with restricted classes of perturbations.
- A method for testing robustness with respect to given perturbations.
- Some small classes of perturbations requiring a full internal model.
- Methods for controller construction