Periodic Output Regulation of Infinite-Dimensional Systems

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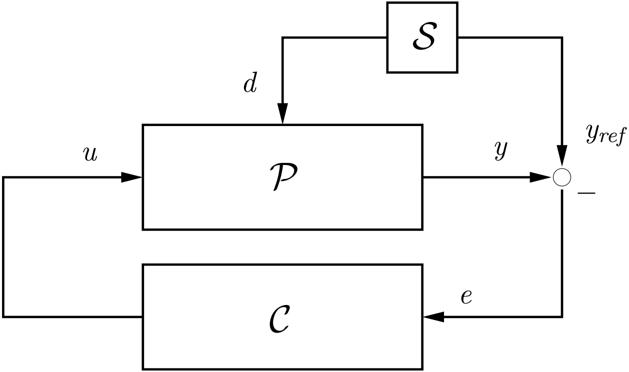
Introduction



Consider a *System* $\mathcal{P}$, a *Signal Generator* $\mathcal{S}$ and a *Controller* $\mathcal{C}$ where y is the *Output* and e the *Regulation Error*.

The Main Problem

Definition (The periodic signal generator $\mathcal{S}$)

The exosystem is of form

$$\begin{aligned} \dot{w}(t) &= S(t)w(t), \qquad w(0) \in W = \mathbb{C}^q, \\ d(t) &= E_d(t)w(t) \\ y_{ref}(t) &= -F_r(t)w(t) \end{aligned}$$

$S \in C_T^1(\mathbb{R}, \mathcal{L}(W))$, $E_d \in C_T^1(\mathbb{R}, \mathcal{L}(W, X))$, $F_r \in C_T^1(\mathbb{R}, \mathcal{L}(W, Y))$.

Here

$$C_T^1(\mathbb{R}, X) = \left\{ f \in C^1(\mathbb{R}, X) \mid f(t + T) = f(t) \text{ for all } t \in \mathbb{R} \right\}.$$

The Main Problem

Definition (Equivalent, Floquet-Lyapunov Theory)

The exosystem is of form

$$\begin{aligned} \dot{v} &= Sv, \qquad v(0) \in W = \mathbb{C}^q, \\ d(t) &= E(t)v(t) \\ y_{ref}(t) &= -F(t)v(t) \end{aligned}$$

$S \in \mathcal{L}(W)$, $E \in C^1_T(\mathbb{R}, \mathcal{L}(W, X))$ and $F \in C^1_T(\mathbb{R}, \mathcal{L}(W, Y))$.

Here

$$C^1_T(\mathbb{R}, X) = \{ f \in C^1(\mathbb{R}, X) \mid f(t+T) = f(t) \text{ for all } t \in \mathbb{R} \}.$$

The Plant $\mathcal{P}$

We consider a plant

$$\begin{aligned} \dot{x} &= Ax + Bu + E(t)v\,, \qquad x(0) \in X \\ e &= Cx + Du + F(t)v \end{aligned}$$

- X is Banach
- A generates an analytic semigroup
- B, C and D are bounded linear operators.

The Plant $\mathcal{P}$

We consider a plant

$$\dot{x} = Ax + Bu + \overbrace{E(t)v}^{d(t)}, \qquad x(0) \in X$$
$$e = Cx + Du + \underbrace{F(t)v}_{-y_{ref}(t)}$$

- X is Banach
- A generates an analytic semigroup
- B, C and D are bounded linear operators.

The Controller $\mathcal{C}$

The feedback controller $(\mathcal{G}_1, \mathcal{G}_2, K)$ is of form

$$\begin{aligned} \dot{z} &= \mathcal{G}_1(t)z + \mathcal{G}_2(t)e, \qquad z(0) \in Z \\ u &= K(t)z. \end{aligned}$$

- Z is Banach
- The family $(\mathcal{G}_1(t), \mathcal{D}(\mathcal{G}_1(t)))$ of operators is T-periodic
- $\mathcal{G}_2 \in C_T^1(\mathbb{R}, \mathcal{L}(Y, Z))$ and $K \in C_T^1(\mathbb{R}, \mathcal{L}(Z, U))$.

The Closed-Loop System

The closed-loop system with $x_e = (x, z)^T$ is given by

$$\begin{aligned} \dot{x}_e &= A_e(t)x_e + B_e(t)v \\ e &= C_e(t)x_e + D_e(t)v. \end{aligned}$$

Assume there exists a *parabolic evolution family* $U_e(t, s)$ associated to the family $(A_e(t), \mathcal{D}(A_e(t)))$.

The Evolution Family $U_e(t,s)$

Definition (A Strongly Continuous Evolution Family)

1. $U_e(t,t) = I$
2. $U_e(t,r)U_e(r,s) = U_e(t,s)$ for $s \leq r \leq t$
3. $\{\,(t,s) \mid t \geq s\,\} \ni (t,s) \mapsto U_e(t,s)$ is strongly continuous.

In the finite-dimensional case: $U_e(t,s) = e^{\int_s^t A_e(r)\,dr}$.

If $A_e(t) \equiv A_e$, generates a semigroup: $U_e(t,s) = T_e(t-s)$.

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The state of the closed-loop system

$$x_e(t) = U_e(t,0)x_e(0) + \int_0^t U_e(t,s)B_e(s)v(s)ds.$$

The Evolution Family $U_e(t,s)$

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Parabolic EF: Satisfied, e.g., if $A_e(t) = A_{sg} + A_b(t)$ where

- A_{sg} generates an analytic semigroup
- $(A_b(t)) \subset \mathcal{L}(X_e)$ and $A_b(\cdot)x_e \in C_T^1(\mathbb{R}, X_e)$ for all $x_e \in X_e$.

The Periodic Output Regulation Problem

Problem (Periodic Output Regulation Problem)

Choose the controller $(\mathcal{G}_1, \mathcal{G}_2, K)$ *such that*

- *The CL system is exponentially stable, i.e. there exist* $M_e, \omega_e > 0$ *s.t.*

$$\|U_e(t,s)\| \le M_e e^{-\omega_e(t-s)}, \qquad t \ge s$$

- *For all initial states* $x_e(0) \in X_e$ *and* $v(0) \in W$ *the regulation error satisfies*

$$e(t) \longrightarrow 0$$

as $t \to \infty$*.*

Earlier Work

- Zhen & Serrani: *The linear periodic output regulation problem*, 2006 ($\dim X = n$)
- Hämäläinen, Pohjolainen, LP ($\dim X = \infty$, auton. exo).

Main Results

Theorem (Characterization of solvability of the PORP)

Assume the controller $(\mathcal{G}_1, \mathcal{G}_2, K)$ *stabilizes the closed-loop system exponentially.*

- *The periodic Sylvester differential equation*

$$\dot{\Sigma}(t) + \Sigma(t)S = A_e(t)\Sigma(t) + B_e(t)$$

  has a unique periodic strong solution $\Sigma_\infty(\cdot)$*.*

- *The controller solves the PORP if and only if this solution satisfies*

$$C_e(t)\Sigma_\infty(t) + D_e(t) = 0$$

  for all $t \in [0, T]$*.*

Controller Design

Theorem (Assumptions)

Assume there exist

- $K_1 \in \mathcal{L}(X,U)$ *such that* $A+BK_1$ *is exp. stable*
- $L(\cdot) \in C_T^1(\mathbb{R}, \mathcal{L}(Y, X_e))$ *such that*

$$\begin{bmatrix} A & E(t) \\ & S \end{bmatrix} - L(t) \begin{bmatrix} C & F(t) \end{bmatrix}$$

  is exponentially stable
- $X(\cdot) \in C_T^1(\mathbb{R}, \mathcal{L}(W,X))$ *and* $U(\cdot) \in C_T^1(\mathbb{R}, \mathcal{L}(W,U))$ *s.t.*

$$\begin{aligned} \dot{X}(t) + X(t)S &= AX(t) + BU(t) + E(t) \\ 0 &= CX(t) + DU(t) + F(t). \end{aligned}$$

Controller Design

Theorem (Choices of Parameters)

The PORP is solved by a controller $(\mathcal{G}_1, \mathcal{G}_2, K)$ *if we choose*

$$K(t) = \begin{bmatrix} K_1 & K_2(t) \end{bmatrix}, \quad K_2(t) = U(t) - K_1 X(t)$$

$$\mathcal{G}_1(t) = \begin{bmatrix} A & E(t) \\ & S \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} K(t) - L(t) \left(\begin{bmatrix} C & F(t) \end{bmatrix} + DK(t) \right)$$

and $\mathcal{G}_2(t) = L(t)$.

Example: A Stable SISO System

Example

- $\dim W = \dim Y = \dim U = 1$
- A exponentially stable, $D \neq 0$
- The exosystem:

$$\dot{v} = 0, \quad E(t) \equiv 0, \quad F(t) \neq 0 \;\; \forall t \in [0, T]$$

Choose $L(t) = (0, 1/F(t))^T$. Then

$$\begin{bmatrix} A & E(t) \\ & S \end{bmatrix} - L(t) \begin{bmatrix} C & F(t) \end{bmatrix} = \begin{bmatrix} A & \\ -1/F(t) \cdot C & -1 \end{bmatrix}$$

is exponentially stable.

Example: A Stable SISO System

Example

Now

$$\begin{aligned} \dot{X}(t) + X(t)S &= AX(t) + BU(t) + E(t) \\ 0 &= CX(t) + DU(t) + F(t) \end{aligned}$$

implies $U(t) = -D^{-1}(CX(t) + F(t))$. Substituting, we have

$$\dot{X}(t) = (A - BD^{-1}C)X(t) - D^{-1}F(t),$$

which has a unique periodic solution if $A - BD^{-1}C$ is exp. stable (satisfied for large enough D).

Conclusion

In this presentation:

- Periodic Output Regulation Problem for infinite-dimensional systems
- Main results
  - Characterization of controllers solving the PORP
  - Construction of an observer-based controller.

Further research topics:

- Solvability of the periodic Sylvester differential equation
- Stabilizability of nonautonomous infinite-dimensional systems.