# A Simple Robust Controller for Port–Hamiltonian Systems

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# Main Objectives

## Problem

*Study the **robust output regulation problem** for infinite-dimensional port-Hamiltonian systems.*

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**Objective:** Design a controller such that the output $y(t)$ of the system converges to a reference signal

$$\|y(t) - y_{ref}(t)\| \to 0, \qquad \text{as} \quad t \to \infty$$

despite disturbance signals $w_{dist}(t)$.

The controller is required to be **robust** in the sense that it tolerates small perturbations in the parameters of the systems.

- Tolerance to **uncertainty** in models
- **Approximate** controller parameters!

# Main Objectives

## Problem

*Study the **robust output regulation problem** for infinite-dimensional port–Hamiltonian systems.*

**Main objectives and results:**

- Robust finite-dimensional controller design
- Focus on **impedance passive** systems and utilize their special properties.
- Use Lyapunov-approach in the study of stability.
- Application to periodic output tracking for a Piezoelectric tube model.

# The Infinite-Dimensional System

Consider a system

$$\dot{x}(t) = (J-R)Qx(t) + Bu(t) + B_d w_{\textit{dist}}(t),$$
$$y(t) = B^* Q x(t)$$

In this presentation

- $X$ is the energy space (Hilbert) with norm $\|x\|_Q$, $Q > 0$
- $J^*Q = -QJ$, $R \geq 0$.
- $u(t), y(t) \in U$ input and output, $w_{\textit{dist}}(t) \in U_d$ disturbance
- Assume $B \in \mathcal{L}(U, X)$ and $\dim U = p < \infty$

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Main assumptions:

- The system is **impedance passive**, i.e.,

$$\frac{1}{2}\frac{d}{dt}\|x(t)\|_Q^2 \leq \mathrm{Re}\langle u(t), y(t)\rangle$$

- The system is exponentially stable (or stabilizable with output feedback).
- The transfer function is denoted by $P(\cdot)$.

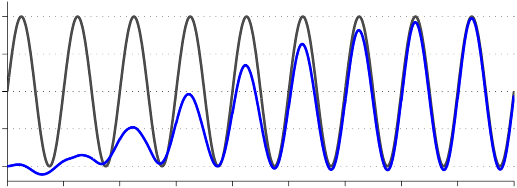
## Problem (The Robust Output Regulation Problem)

*Choose a control law in such a way that*

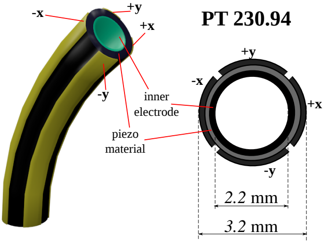
- *The output* $y(t)$ *tracks a given reference signal* $y_{\textit{ref}}(t)$ *asymptotically, i.e.*

  $$\|y(t) - y_{\textit{ref}}(t)\| \to 0 \qquad \textit{as } t \to \infty$$

  *for all initial states* $x_0 \in X$*.*
- *The above property is robust with respect to "small" perturbations in the operators* $(J, R, Q, B, B_d)$ *of the system.*



# Piezoelectric Tube Model




Robust tracking for a piezoelectric tube

- Motivation from atomic force microscopy
- Modeled with a 1D Timoshenko beam with frictional damping
- Distributed collocated control and observation
- Periodic tracking of the angular momentum near the free end of the tube.

# Earlier Work

Robust regulation for port-Hamiltonian and passive systems:

- Rebarber–Weiss '03
  - A "simple" robust controller for **passive** well-posed systems
- Humaloja–LP '18 & Humaloja–Kurula-LP Arxiv Nov '17
  - Robust regulation of **port-Hamiltonian** and boundary control systems
- LP Arxiv Jun '17
  - Robust regulation for **passive** regular linear systems
- Internal Model Principle for $\infty$-dimensional systems (LP '10, '14)

For other classes of infinite-dimensional systems:

- Hämäläinen-Pohjolainen '96, '00, '10, Logemann-Townley '97 Weiss-Häfele '99, Immonen-Pohjolainen '06, Boulite *et. al.* '09, LP '16,'17.

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In this work:

**Problem**

*Use the state-space approach and Lyapunov techniques in construction and analysis of a robust port-Hamiltonian controller.*

# Comments

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*Use the state-space approach and Lyapunov techniques in construction and analysis of a robust port-Hamiltonian controller.*

Significance of the assumptions and chosen approach:

- pH systems — A natural framework for modeling mechanical systems with 1D PDEs
- Passivity — Leads to especially nice study of stability
- Lyapunov approach — Lays a foundation for treatment of nonlinearities

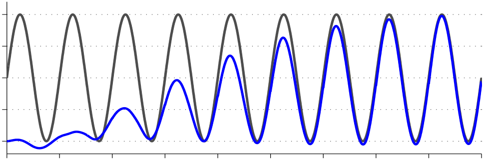
# The Reference Signals

The reference signals we consider are of the form

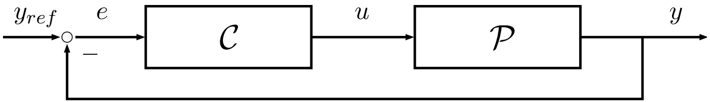
$$y_{ref}(t) = a_0 + \sum_{k=1}^{q} \left[ a_k^1 \cos(\omega_k t) + a_k^2 \sin(\omega_k t) \right],$$

$$w_{dist}(t) = b_0 + \sum_{k=1}^{q} \left[ b_k^1 \cos(\omega_k t) + b_k^2 \sin(\omega_k t) \right],$$

with **frequencies** $\{\omega_k\}_{k=0}^{q} \subset \mathbb{R}_+$, unknown amplitudes and phases.



# The Port-Hamiltonian Error Feedback Controller




We consider an error feedback controller $(J_c, B_c, B_c^*)$ of the form

$$\begin{aligned}\dot{x}_c(t) &= J_c x_c(t) + B_c(y_{ref}(t) - y(t)),\\ u(t) &= B_c^* x_c(t)\end{aligned}$$

where $J_c \in \mathbb{R}^q$ and $J_c^* = -J_c$, and $B_c \in \mathcal{L}(U, X_c)$.

# Controller Construction

Choose the parameters of the **internal model based** controller

$$\dot{x}_c(t) = J_c x_c(t) + B_c(y_{\mathit{ref}}(t) - y(t)),$$
$$u(t) = B_c^* x_c(t)$$

so that

$$J_c = \begin{bmatrix} J_0 & & \\ & \ddots & \\ & & J_q \end{bmatrix}, \quad \text{where} \quad J_k = \begin{bmatrix} 0 & \omega_k I_U \\ -\omega_k I_U & 0 \end{bmatrix}, \; J_0 = 0 I_U$$

and $B_c$ is such that $(J_c, B_c)$ is controllable.

# Main Result

## Theorem

*Assume $P(i\omega_k)$ is nonsingular for every $k$. The port-Hamiltonian error feedback controller solves the robust output regulation problem provided that $\|B_c\|$ is chosen to be sufficiently small.*

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## Proof.

Due to the **Internal Model Principle**, we require two things:

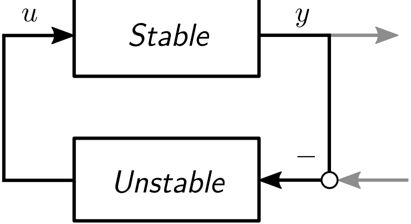
Step 1° Include a suitable internal model into the controller — **guaranteed by the chosen controller structure**

Step 2° Use the rest of the controller's parameters to stabilize the closed-loop system — Analyse stability with the Lyapunov approach, and utilize passivity.

□

# Comments on the Stability Proof

- The Lyapunov approach is a natural choice for analysing the stability of coupled pH-systems.
- The closed-loop system has the form of a **stable-unstable** coupling through **power-preserving interconnection**. This case was analysed in Ramirez-Le Gorrec-Macchelli-Zwart '14, but the situations have a crucial difference (the proof does not translate).



# Comments on the Stability Proof

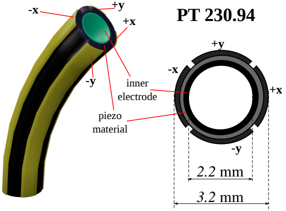
- The current proof annoyingly gives a suboptimal result, as it's known from frequency domain analysis (Rebarber-Weiss '03) that the low-gain condition "$\|B_c\|$ small" is not necessary.
- In particular, the frequency domain techniques help in guarateeing suitable high-frequency behaviour easily.
- On the other hand Lyapunov techniques are beneficial for extending the study for situations with **nonlinearities**.

# Example

1D Timoshenko beam with observation of angular momentum

$$y(t) = B^*Qx(t) = \int_{4/5}^{1} \frac{\partial \phi}{\partial t}(\xi, t)d\xi$$

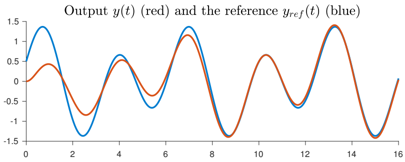
and the corresponding control input.

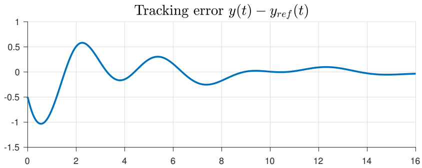


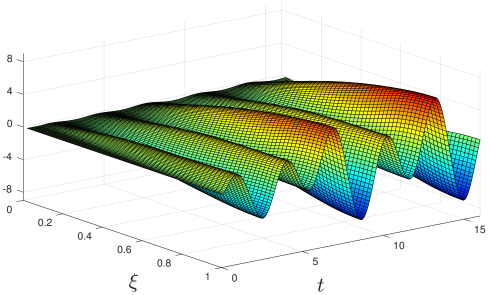

To achieve tracking of signals with frequencies $\pm i, \pm 2i$, choose

$$J_c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 \end{bmatrix}, \quad B_c = \gamma_c \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \gamma_c > 0.$$

# Reference $y_{ref}(t) = \frac{1}{2}\cos(t) + \sin(2t)$



# Conclusions

In this presentation:

- Simple controller for robust output tracking of passive port–Hamiltonian systems
- Lyapunov approach for analysis of the closed-loop stability
- Concrete case: Trajectory tracking for a piezoelectric tube