Saturated Output Regulation of Distributed Parameter Systems with Collocated Actuators and Sensors

Thavamani Govindaraj, Jukka-Pekka Humaloja, Lassi Paunonen

Tampere University (Finland) and University of Alberta (Canada)

IFAC World Congress, July 9–14, 2023

Outline

- In reality, it is natural to consider limitations in the actuators.
- Linear systems subject to input saturation

$$\dot{x}(t) = Ax(t) + B\phi(u(t)) + B_d w_d(t), \quad x(0) = x_0,$$
$$y(t) = Cx(t).$$

Here ϕ is a saturation function where the input $u(t)$ takes values in an interval $[u_{min}, u_{max}]$.

- Output regulation of infinite-dimensional linear systems subject to input saturation:

For given $y_{\mathrm{ref}}(t)$ and $w_d(t)$, construct $u(t)$ so that

$$\|y(t) - y_{\mathrm{ref}}(t)\| \to 0, \qquad \textit{as} \quad t \to \infty$$

for all $x_0 \in X$.

We consider class of abstract SISO systems of the form

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B\phi(u(t)) + B_d w_d(t), \quad x(0) = x_0,\\ y(t) &= B^* x(t)\end{aligned}$$

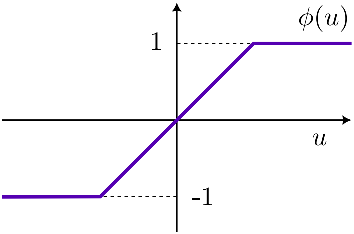
on a real Hilbert space X. Here $x(t) \in X$ is the state, $u(t) \in \mathbb{R}$ the input, $y(t) \in \mathbb{R}$ the output, and $w_d(t) \in \mathbb{R}^{n_d}$ an external disturbance.

We consider class of abstract SISO systems of the form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B\phi(u(t)) + B_d w_d(t), \quad x(0) = x_0, \\ y(t) &= B^* x(t) \end{aligned}$$

on a real Hilbert space X. Here $x(t) \in X$ is the state, $u(t) \in \mathbb{R}$ the input, $y(t) \in \mathbb{R}$ the output, and $w_d(t) \in \mathbb{R}^{n_d}$ an external disturbance. The saturation function ϕ is defined as

$$\phi(u) = \begin{cases} u, & |u| \le 1 \\ 1, & u > 1 \\ -1, & u < -1. \end{cases}$$



Comments

- The considered class of systems arise in the study of systems with collocated actuators and sensors.
- Output regulation has been studied actively for finite-dimensional saturated systems. For DPS, results mainly focus on PI control (constant $y_{\text{ref}}(t)$ and $w_d(t)$), esp. [Logemann–Ryan–Townley '98–, Logemann et. al., '00 Oostveen '00]
- We consider time-varying $y_{\text{ref}}(t)$ and $w_d(t)$. Our results extend the finite-dimensional theory in [Saberi et al., 2003].

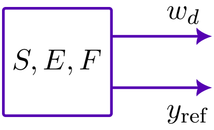
The Reference and Disturbance Signals

We consider references $y_{\text{ref}}(t)$ and disturbances $w_d(t)$ of the forms

$$y_{\text{ref}}(t) = \sum_{k=1}^{q_0} a_k \cos(\omega_k + \theta_k), \qquad w_d(t) = \sum_{k=1}^{q_0} b_k \cos(\omega_k + \varphi_k)$$

for **known** frequencies ω_k, amplitudes a_k, b_k, and phases θ_k, φ_k. Such $y_{\text{ref}}(t)$ and $w_d(t)$ can be generated by **an exosystem**

$$\begin{aligned} \dot{v}(t) &= Sv(t), \quad v(0) = v_0 \in \mathbb{R}^q, \\ w_d(t) &= Ev(t), \\ y_{\text{ref}}(t) &= -Fv(t) \end{aligned}$$



where $S \in \mathbb{R}^{q\times q}$, $F \in \mathbb{R}^{1\times q}$, $E \in \mathbb{R}^{n_d\times q}$, and v_0 are determined by ω_k, a_k, b_k, θ_k, and φ_k. In particular, $\sigma(S) = \{\pm i\omega_k\}_{k=1}^{q}$.

Semi-Global Output Regulation Problem.

Consider a compact set $\mathcal{W}_0 \subset \mathbb{R}^q$. Find a linear control law in the form

$$u(t) = -\kappa y(t) + Lv(t)$$

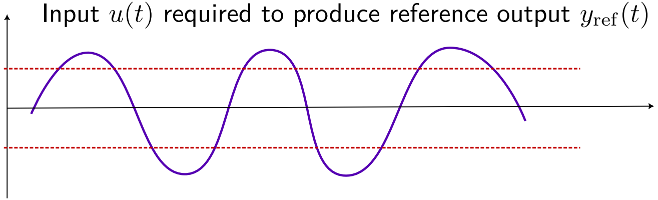
such that $\kappa > 0,\ L \in \mathbb{R}^{1\times q}$ and

1. $0 \in X$ is a globally asymptotically stable equilibrium of system
$$\dot{x}(t) = Ax(t) + B\phi(-\kappa y(t)), \qquad x(0) = x_0.$$
2. For all $x_0 \in X$ and $v_0 \in \mathcal{W}_0$, the error between the output $y(t)$ and the reference signal $y_{ref}(t)$ satisfies
$$\lim_{t\to\infty} |y(t) - y_{ref}(t)| = 0.$$

Comment: $u(t)$ is based on output feedback + $y_{\mathrm{ref}}(t)$ and $w_d(t)$.

Why "Semi-global"?

"For all $x_0 \in X$ and $v_0 \in \mathcal{W}_0$, the error between the output $y(t)$ and the reference signal $y_{ref}(t)$ satisfies $|y(t) - y_{ref}(t)| \to 0$."



- The problem is unsolvable for $y_{\rm ref}$ and w_d with large amplitudes, which corresponds to large $\|v_0\|$!
- No limitations for x_0.

Main Assumptions

$$\dot{x}(t) = Ax(t) + B\phi(u(t)) + B_d w_d(t), \quad x(0) = x_0,$$
$$y(t) = B^* x(t)$$

Assumption 1

We assume

- *The operator A generates a C_0-semigroup $T(t)$ of contractions on X,*
- $B \in \mathcal{L}(\mathbb{R}, X)$ *and* $B_d \in \mathcal{L}(\mathbb{R}^{n_d}, X)$*,*
- *The operator $A - \kappa BB^*$ generates a **strongly stable** contraction semigroup $T_{-\kappa BB^*}(t)$ for any $\kappa > 0$.*

Theorem 2

Consider the compact set $\mathcal{W}_0 \subset \mathbb{R}^q$. Under the given Assumption 1, the semi-global output regulation problem is solvable if there exist $\Pi \in \mathcal{L}(\mathbb{R}^q, X)$ with $\mathcal{R}(\Pi) \subset D(A)$ and $\Gamma \in \mathbb{R}^{1\times q}$ such that they solve the regulator equations

$$\Pi S = A\Pi + B\Gamma + B_d E$$
$$0 = B^*\Pi + F$$

and there exists a $\delta > 0$ such that $\sup_{t\geq 0} |\Gamma v(t)| \leq 1 - \delta$ for all $v(t) = e^{St}v_0$ with $v_0 \in \mathcal{W}_0$. In this case, for any $\kappa > 0$ the feedback law

$$u(t) = -\kappa y(t) + (\kappa B^*\Pi + \Gamma)v(t)$$

solves the semi-global output regulation problem.

- Our main result is an infinite-dimensional generalization of a result of [Saberi et al., 2003].
- Our approach for showing the asymptotic convergence of the regulation error is motivated by the techniques in [Curtain and Zwart, 2020].

Outline of the Proof:

- By using strong stabilizability of the system by negative output feedback and uniform Lipschitz continuity of the saturation function, we have from [Curtain and Zwart, 2016] that the origin of the system $\dot{x}(t) = Ax(t) + B\phi(-\kappa y(t))$, $x(0) = x_0$ is globally asymptotically stable.
- We introduce a variable $\xi(t) = x(t) - \Pi v(t)$ which is the mild solution of

$$\dot{\xi}(t) = A\xi(t) + B[\phi(-\kappa B^*\xi(t) + \Gamma v(t)) - \Gamma v(t)]$$
$$\xi(0) = \xi_0.$$

on X.

By using the assumptions of the theorem, we can show that $\xi(t) = x(t) - \Pi v(t)$ decays to zero asymptotically. This implies

$$\begin{aligned} y(t) - y_{ref}(t) &= B^* x(t) - y_{ref}(t) \\ &= \underbrace{B^* \xi(t)}_{\to 0} + \underbrace{(B^* \Pi + F) v(t)}_{=0} \to 0 \end{aligned}$$

as $t \to \infty$ due to the fact that Π satisfies $B^* \Pi + F = 0$ by assumption.

Lemma 3 ([Byrnes et al., 2000])

Suppose Assumption 1 holds. The regulator equations are solvable if and only if $\pm i\omega_k$ *do not coincide with the transmission zeros of the system* (A, B, B^*).

- Equivalently, $G(\pm i\omega_k) \neq 0$ for all k, where $G(\cdot) = B^*(\cdot I - A)^{-1}B$ is the transfer function of the system (A, B, B^*).
- In this case, Π and Γ have explicit formulas determined by

$$\Pi\Phi_k = (i\mu_k - A)^{-1}(B\Gamma + B_d E)\Phi_k$$
$$\Gamma\Phi_k = -G(i\mu_k)^{-1}(B^*(i\mu_k - A)^{-1}B_d E + F)\Phi_k,$$

$k = 1, 2, \cdots, q$, where $i\mu_k$ and Φ_k are the eigenvalues and the corresponding orthonormal eigenvectors of S

1. Main Problem
2. Results
3. Example

We consider a dynamic model of a flexible satellite [Bontsema et al., 1988, He and Ge, 2015]

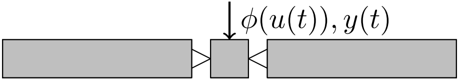


Figure: A flexible satellite model

$$\begin{aligned}
\ddot{w}_l(\xi,t) + w_l''''(\xi,t) + 5\dot{w}_l(\xi,t) &= 0,\ -1 < \xi < 0, t > 0,\\
\ddot{w}_r(\xi,t) + w_r''''(\xi,t) + 5\dot{w}_r(\xi,t) &= 0,\ 0 < \xi < 1, t > 0,\\
\ddot{w}_c(t) &= w_l'''(0,t) - w_r'''(0,t) + \boldsymbol{\phi(u(t))} + w_d(t),\\
\ddot{\theta}_c(t) &= -w_l''(0,t) + w_r''(0,t).
\end{aligned}$$

with measured output

$$y(t) = \dot{w}_c(t).$$

The boundary conditions are given by

$$\begin{aligned} w_l''(-1,t) = 0, \quad & w_l'''(-1,t) = 0, \\ w_r''(1,t) = 0, \quad & w_r'''(1,t) = 0, \\ \dot{w}_l(0,t) = \dot{w}_r(0,t) &= \dot{w}_c(t), \\ \dot{w}_l'(0,t) = \dot{w}_r'(0,t) &= \dot{\theta}_c(t). \end{aligned}$$

- The satellite model can be written in the form [Govindaraj et al., 2023]

$$\dot{x}(t) = Ax(t) + B\phi(u(t)) + B_d w_d(t), \quad x(0) = x_0,$$
$$y(t) = B^* x(t)$$

where the operator A generates an exponentially stable contraction semigroup on the state space $X = L^2(-1, 0; \mathbb{R}^2) \times L^2(0, 1; \mathbb{R}^2) \times \mathbb{R}^2$.

- It can be also verified that $A - \kappa BB^*$ generates an exponentially stable contraction semigroup on X for any $\kappa > 0$ [Govindaraj et al., 2020].

Our goal is to track the reference signal $y_{ref}(t) = 0.09 \sin(1.5t)$ and reject the disturbance $w_d(t) \equiv 0.08$. We choose the exosystem

$$\dot{v}(t) = \begin{bmatrix} 0 & 1.5 & 0 \\ -1.5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} v(t), \quad v(0) = \begin{bmatrix} 0 \\ 0.09 \\ 0.08 \end{bmatrix},$$
$$y_{ref}(t) = -\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} v(t),$$
$$w_d(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} v(t).$$

- The eigenvalues are given by $\{0, \pm 1.5i\}$.
- Moreover, the satellite system does not have any transmission zeros at 0, $1.5i$ and $-1.5i$ [Govindaraj et al., 2023]

The control parameters Γ and Π can be obtained by using

$$\Pi\Phi_k = (i\mu_k - A)^{-1}(B\Gamma + B_d E)\Phi_k$$
$$\Gamma\Phi_k = -G(i\mu_k)^{-1}(B^*(i\mu_k - A)^{-1}B_d E + F)\Phi_k,$$

and the formulas lead to

$$\Gamma v(t) = 0.09|G(1.5i)^{-1}|\sin(1.5t + \theta) + 0.08,$$
$$\theta = \tan^{-1}(\beta/\alpha), \alpha = \mathsf{Re}(G(i\omega)), \beta = -\mathsf{Im}(G(i\omega))$$
$$B^*\Pi v(t) = 0.09\sin(1.5t)$$

in the control law

$$u(t) = -\kappa y(t) + (\kappa B^*\Pi + \Gamma)v(t)$$

where $G(\cdot)$ is the transfer function of the satellite system.

Output tracking is achieved using the control input
$u(t) = -\kappa y(t) + (\kappa B^*\Pi + \Gamma)v(t)$.

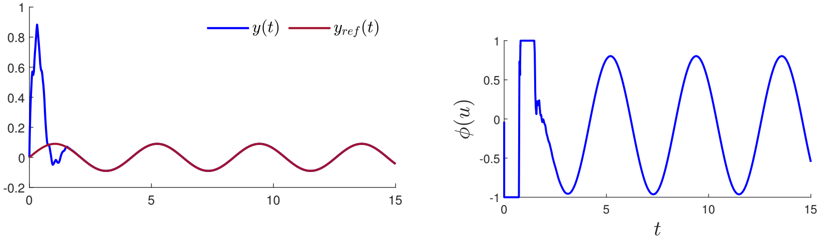


Figure: The output $y(t)$ (blue), reference $y_{\text{ref}}(t)$ (red), and the saturated control input $\phi(u(t))$ (right) for $\kappa = 100$.

Conclusion

- We considered class of strongly stabilizable infinite-dimensional linear systems with collocated actuators and sensors subject to input saturation.
- We proposed a linear output feedback control law that solves the semi-global output regulation problem.
- The results are illustrated with an example of a flexible satellite model subject to input saturation.

Bontsema, J., Curtain, R. F., and Schumacher, J. M. (1988).
Robust control of flexible structures a case study.
Automatica, 24(2):177 – 186.

Byrnes, C. I., Laukó, I. G., Gilliam, D. S., and Shubov, V. I. (2000).
Output regulation for linear distributed parameter systems.
IEEE Trans. Automat. Control, 45(12):2236–2252.

Curtain, R. and Zwart, H. (2016).
Stabilization of collocated systems by nonlinear boundary control.
Systems & Control Letters, 96:11–14.

Curtain, R. F. and Zwart, H. (2020).
Introduction to Infinite-Dimensional Systems Theory: A State-Space Approach, volume 71 of *Texts in Applied Mathematics*.
Springer, New York.

Govindaraj, T., Humaloja, J.-P., and Paunonen, L. (2020).
Robust output regulation of a flexible satellite.
IFAC-PapersOnLine, 53(2):7795–7800.

Govindaraj, T., Humaloja, J.-P., and Paunonen, L. (2023).
Robust controllers for a flexible satellite model.
Mathematical Control and Related Fields.

He, W. and Ge, S. S. (2015).
Dynamic modeling and vibration control of a flexible satellite.
IEEE Transactions on Aerospace and Electronic Systems, 51(2):1422–1431.

Saberi, A., Stoorvogel, A. A., and Sannuti, P. (2003).
Control of linear systems with regulation and input constraints.

Communications and Control Engineering Series.
Springer-Verlag London, Ltd., London.

Thank You!

Advertisement:

Open Postdoc position at the research group in Tampere, Finland. Application deadline **September 4th, 2023**.

For details, see

https://sysgrouptampere.wordpress.com

or contact lassi.paunonen@tuni.fi