Internal Model Principle for Distributed Parameter Systems

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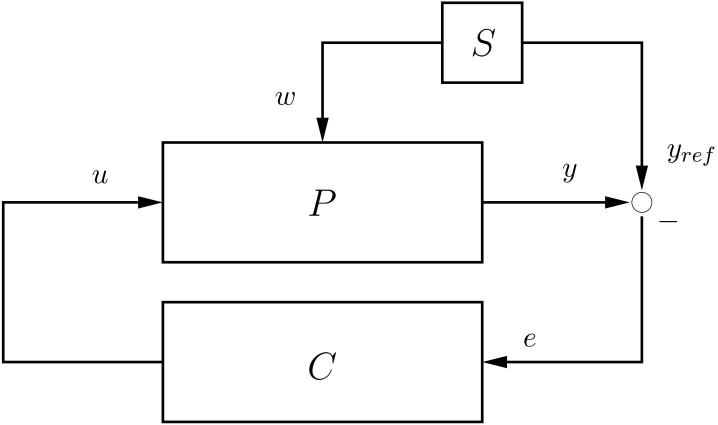
The Internal Model Principle

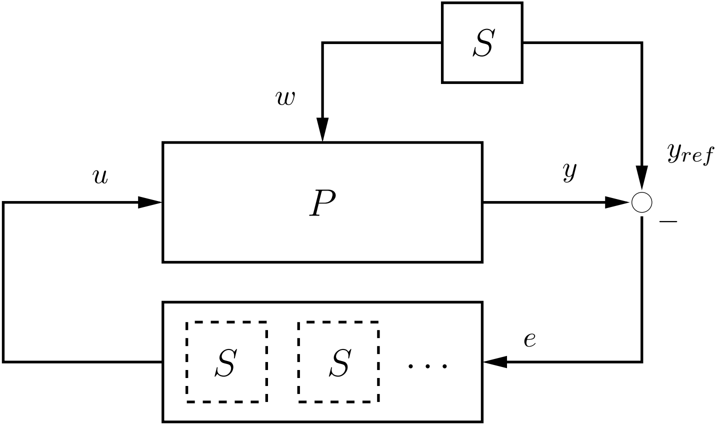
Theorem (Francis & Wonham, 1970's)

A stabilizing feedback controller solves the robust output regulation problem if and only if it incorporates a suitably reduplicated model of the signal generator.

Feedback Controller



The p-Copy Internal Model Principle



Main Problem

Problem

Generalize the p-copy Internal Model Principle for distributed parameter systems and infinite-dimensional signal generators.

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Generalize the p-copy Internal Model Principle for distributed parameter systems and infinite-dimensional signal generators.

(i) Infinite-dimensional signal generator;
(ii) Definition of the p-copy Internal Model;
(iii) The p-copy Internal Model Principle.

Earlier Work

- B. A. Francis & W. M. Wonham - The finite-dimensional Internal Model Principle, 1970's
- M. K. P. Bhat - Extension for distributed parameter systems, 1976
- E. Immonen - Partial extension for infinite-dimensional signal generators, 2006

The System

System $\Sigma(A, B, C, D, E, F)$ on X with regulation error $e \in Y$

$$\begin{aligned} \dot{x} &= Ax + Bu + Ev, \qquad x(0) = x_0 \\ e &= Cx + Du + Fv \end{aligned}$$

The signal generator on W

$$\dot{v} = Sv, \qquad v(0) = v_0,$$

The error feedback controller $(\mathcal{G}_1, \mathcal{G}_2, K)$ on Z

$$\begin{aligned} \dot{z} &= \mathcal{G}_1 z + \mathcal{G}_2 e, \qquad z(0) = z_0 \\ u &= Kz \end{aligned}$$

The Closed-Loop System

The closed-loop system with state $(x(t), z(t))^T \in X \times Z$ is given by

$$\begin{aligned} \dot{x}_e &= A_e x_e + B_e v, \qquad x_e(0) = (x_0, z_0)^T \\ e &= C_e x_e + D_e v \end{aligned}$$

where $C_e = \begin{bmatrix} C & DK \end{bmatrix}$, $D_e = F$,

$$A_e = \begin{bmatrix} A & BK \\ \mathcal{G}_2 C & \mathcal{G}_1 + \mathcal{G}_2 DK \end{bmatrix} \quad \text{and} \quad B_e = \begin{bmatrix} E \\ \mathcal{G}_2 F \end{bmatrix}.$$

Robust Output Regulation Problem

Problem (Robust Output Regulation Problem)

Choose controller parameters $(\mathcal{G}_1, \mathcal{G}_2, K)$ *such that*

(i) *The closed-loop system operator* A_e *generates a strongly stable* C_0*-semigroup on* $X \times Z$*;*

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(ii) *For all initial states* x_0, z_0 *and* v_0 *the regulation error* $e(t)$ *decays to zero as* $t \to \infty$*;*

(iii) *Property (ii) is robust with respect to a class of perturbations preserving the strong stability of the closed-loop system.*

A Signal Generator with Infinite Number of Jordan Blocks

$$\begin{bmatrix} \ddots & & & \\ & \begin{bmatrix} i\omega_{-1} & 1 & 0 \\ & i\omega_{-1} & 1 \\ & & i\omega_{-1} \end{bmatrix} & & & \\ & & \begin{bmatrix} i\omega_0 & 1 \\ & i\omega_0 \end{bmatrix} & & \\ & & & \begin{bmatrix} i\omega_1 & 1 & 0 \\ & i\omega_1 & 1 \\ & & i\omega_1 \end{bmatrix} & \\ & & & & \ddots \end{bmatrix}$$

The System Operator

Define

$$W = \overline{\mathrm{span}}\left\{ \phi_k^l \;\middle|\; k \in \mathbb{Z},\ l = 1, \ldots, n_k \right\}, \qquad \langle \phi_k^l, \phi_n^m \rangle = \delta_{kn}\delta_{lm}.$$

Let $\{i\omega_k\}_{k\in\mathbb{Z}} \subset i\mathbb{R}$ and define "Jordan blocks" $S_k \in \mathcal{L}(W)$ as

$$S_k = i\omega_k \langle \cdot, \phi_k^1 \rangle \phi_k^1 + \sum_{l=2}^{n_k} \langle \cdot, \phi_k^l \rangle \left(i\omega_k \phi_k^l + \phi_k^{l-1} \right)$$

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Define the system operator of the signal generator as

$$Sv = \sum_{k\in\mathbb{Z}} S_k v, \quad \mathcal{D}(S) = \Big\{\, v \in W \;\Big|\; \sum_{k\in\mathbb{Z}} \|S_k v\|^2 < \infty \,\Big\}.$$

Finite-Dimensional p-Copy Internal Model

Let $p = \dim Y$ = dimension of the output space

In classical finite-dimensional control theory:

Definition (p-Copy Internal Model)

A controller $(\mathcal{G}_1,\mathcal{G}_2)$ incorporates a *p-Copy Internal Model* of the exosystem S if

whenever $s \in \sigma(S)$ is an eigenvalue of S such that $d(s)$ is the dimension of the largest Jordan block associated to s, then

- $s \in \sigma(\mathcal{G}_1)$ and
- $\mathcal{G}_1$ has at least p Jordan blocks of dimension greater or equal to $d(s)$ associated to s.

Infinite-Dimensional p-Copy Internal Model

For our infinite-dimensional exosystem $\dot{v} = Sv$:

$\sigma(S) = \sigma_p(S) = \{i\omega_k\}_k$

$d_k :=$ dimension of the largest Jordan block S_k associated to $i\omega_k$.

Definition (p-Copy Internal Model)

A controller $(\mathcal{G}_1,\mathcal{G}_2)$ incorporates a *p-Copy Internal Model* of the exosystem S if for all $k \in \mathbb{Z}$

- $i\omega_k \in \sigma_p(\mathcal{G}_1)$ and
- $\mathcal{G}_1$ has at least p independent Jordan chains of length greater or equal to d_k associated to the eigenvalue $i\omega_k$.

The Internal Model Principle

Theorem (p-Copy Internal Model Principle)

Let $\dim Y < \infty$ *and* $\sigma(A_e) \cap \sigma(S) = \varnothing$. *The controller* $(\mathcal{G}_1, \mathcal{G}_2)$ *solves the robust output regulation problem if and only if for all* $k \in \mathbb{Z}$ *we have*

- $i\omega_k \in \sigma_p(\mathcal{G}_1)$
- $\mathcal{G}_1$ *has at least* $\dim Y$ *independent Jordan chains of length greater or equal to* d_k *associated to the eigenvalue* $i\omega_k$.

Outline of the Proof

Robust Regulation

$\Updownarrow$

Internal Model Structure

$\Updownarrow$

$\mathcal{G}$-Conditions

$\Updownarrow$

p-Copy Internal Model

Internal Model Structure

Theorem (E. Immonen)

The controller $(\mathcal{G}_1, \mathcal{G}_2)$ solves the robust output regulation problem if and only if it has Internal Model Structure, i.e.

$$\forall \Lambda, \Delta : \qquad \Lambda S = \mathcal{G}_1 \Lambda + \mathcal{G}_2 \Delta \quad \Rightarrow \quad \Delta = 0, \qquad \text{(IMS)}$$

where $\Lambda \in \mathcal{L}(W, Z)$ is such that $\Lambda(\mathcal{D}(S)) \subset \mathcal{D}(\mathcal{G}_1)$ and $\Delta \in \mathcal{L}(W, Y)$.

$\mathcal{G}$-Conditions

Theorem (LP, S. Pohjolainen)

Let $\sigma(A_e) \cap \sigma(S) = \varnothing$. The controller $(\mathcal{G}_1, \mathcal{G}_2)$ has Internal Model Structure if and only if the following $\mathcal{G}$-conditions are satisfied:

$$\mathcal{R}(i\omega_k I - \mathcal{G}_1) \cap \mathcal{R}(\mathcal{G}_2) = \{0\}, \qquad \forall k \in \mathbb{Z}$$

$$\mathcal{N}(\mathcal{G}_2) = \{0\}$$

and

$$\mathcal{N}(i\omega_k I - \mathcal{G}_1)^{d_k - 1} \subset \mathcal{R}(i\omega_k I - \mathcal{G}_1) \quad \forall k \in \mathbb{Z}.$$

p-Copy Internal Model

Theorem (LP, S. Pohjolainen)

Let $\dim Y < \infty$ and $\sigma(A_e) \cap \sigma(S) = \varnothing$. The controller $(\mathcal{G}_1, \mathcal{G}_2)$ satisfies the $\mathcal{G}$-conditions if and only if for all $k \in \mathbb{Z}$ we have

- $i\omega_k \in \sigma_p(\mathcal{G}_1)$
- *$\mathcal{G}_1$ has at least $\dim Y$ independent Jordan chains of length greater or equal to d_k associated to the eigenvalue $i\omega_k$.*

Summary

Robust Regulation

$\Updownarrow$

Internal Model Structure

$\Updownarrow$ $\quad \sigma(A_e) \cap \sigma(S) = \varnothing$

$\mathcal{G}$-Conditions

$\Updownarrow$ $\quad \sigma(A_e) \cap \sigma(S) = \varnothing$, $\dim Y < \infty$

p-Copy Internal Model

Conclusions

In this presentation:

- Generalization of the classical p-copy Internal Model Principle of Francis and Wonham for distributed parameter systems.

Remarks:

- The $\mathcal{G}$-Conditions generalize the p-copy Internal Model and are meaninful in the case $\dim Y = \infty$.

Future research:

- A nonlinear signal generator $\dot{w} = s(w)$.