Robust Output Regulation of Regular Linear Systems

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Main Objectives

Problem

*Study the **robust output regulation problem** for infinite-dimensional systems with unbounded input and output operators.*

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Problem

Study the ***robust output regulation problem*** *for infinite-dimensional systems with unbounded input and output operators.*

Objective: Design a controller such that the output $y(t)$ of the system converges to a reference signal

$$\|y(t) - y_{ref}(t)\| \to 0, \qquad \text{as} \quad t \to \infty$$

despite disturbance signals $w_{dist}(t)$.

The controller is required to be **robust** in the sense that it tolerates small perturbations in the parameters of the system.

Main Objectives

Problem

*Study the **robust output regulation problem** for infinite-dimensional systems with unbounded input and output operators.*

Main results:

- Robust controller design
- Study of closed-loop stability properties.
- Special emphasis on **passive** systems.

The Infinite-Dimensional System

Consider a system

$$\dot{x}(t) = Ax(t) + Bu(t) + B_d w_{dist}(t), \qquad x(0) = x_0 \in X$$
$$y(t) = C_\Lambda x(t) + Du(t)$$

In this presentation

- X is a Banach space
- A generates a semigroup $T(t)$
- $u(t) \in U$ input, $y(t) \in Y$ output, $w_{dist}(t) \in U_d$ disturbance
- Assume $\dim Y = p < \infty$ (for $\dim Y = \infty$ with modifications)
- Let $w_{dist}(t) \equiv 0$ (for simplicity)

The Infinite-Dimensional Plant

Consider a plant

$$\dot{x}(t) = Ax(t) + Bu(t) + B_d w_{dist}(t), \qquad x(0) = x_0 \in X$$
$$y(t) = C_\Lambda x(t) + Du(t)$$

Assumptions on B and C:

- $B \in \mathcal{L}(\mathbb{C}^m, X_{-1})$ and $C \in \mathcal{L}(X_1, \mathbb{C}^p)$ are admissible
- (A, B, C, D) is regular, i.e., for one/all $\lambda \in \rho(A)$

$$P(\lambda) = C_\Lambda R(\lambda, A_{-1})B + D$$

is well-defined and $P(\lambda)u \to Du$ as $\lambda \to \infty$ for all $u \in U$.

Problem (The Robust Output Regulation Problem)

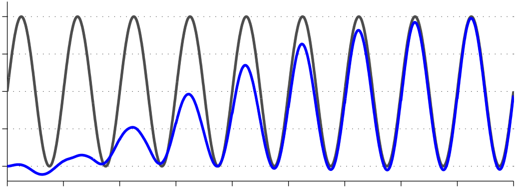
Choose a control law in such a way that

- *The output* $y(t)$ *tracks a given reference signal* $y_{\mathit{ref}}(t)$ *asymptotically, i.e.*

$$\|y(s) - y_{\mathit{ref}}(s)\| \to 0 \qquad \text{as } t \to \infty$$

  for all initial states $x_0 \in X$.

- *The above property is robust with respect to "small" perturbations in the operators* (A, B, B_d, C, D) *of the plant.*



The Reference Signals

The reference signals we consider are of the form

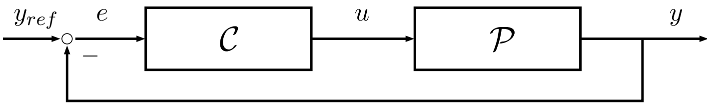
$$y_{ref}(t) = \sum_{k\in\mathcal{I}} y_r^k e^{i\omega_k t}, \qquad (y_r^k)_k \in \ell^1(Y)$$

$y_{ref}(t)$ contains the frequencies $(i\omega_k)_{k\in\mathcal{I}} \in i\mathbb{R}$ (no acc. points).

The set $\mathcal{I} \subset \mathbb{Z}$ may be

- Finite, $\mathcal{I} = \{1, \dots, q\}$, finite number of frequency components
- Infinite, e.g., $\mathcal{I} = \mathbb{Z}$, and $\omega_k = \frac{2\pi k}{\tau}$, τ-periodic functions.
- Infinite, uniformly continuous, almost periodic functions.

The Dynamic Error Feedback Controller



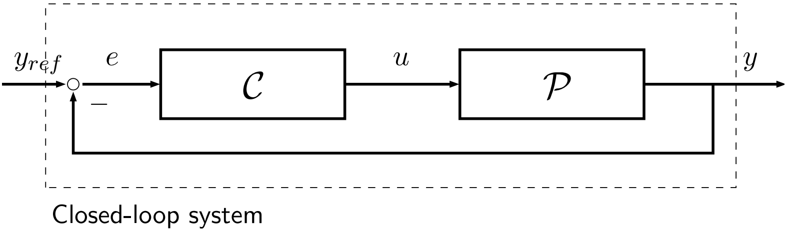
We consider an error feedback controller $(\mathcal{G}_1, \mathcal{G}_2, K)$ of the form

$$\dot{z}(t) = \mathcal{G}_1 z(t) + \mathcal{G}_2(y_{ref}(t) - y(t)) \qquad z(0) = z_0 \in Z,$$
$$u(t) = Kz(t) + D_c(y_{ref}(t) - y(t)),$$

which is a regular linear system on a Banach Z.

D_c is equivalent to output feedback in the plant
$\rightarrow$ can be used in "pre-stabilizing" an unstable system.

The Closed-Loop System



"Closed-loop stability" means that the internal dynamics of the closed-loop system are stable.

The Closed-Loop System

The closed-loop system with state $(x(t), z(t))^T \in X \times Z$ is

$$\begin{aligned} \dot{x}_e(t) &= A_e x_e(t) + B_e y_{\textit{ref}}(t), \qquad x_e(0) = x_{e0} = (x_0, z_0)^T \\ e(t) &= C_e x_e(t) + D_e y_{\textit{ref}}(t). \end{aligned}$$

The closed-loop system is regular (under mild assumptions), A_e generates a semigroup $T_e(t)$.

The Control Problem

Problem (Robust Output Regulation)

Choose a control law in such a way that

- *The closed-loop semigroup $T_e(t)$ is stable.*
- *The output $y(t)$ tracks a given reference signal $y_{ref}(t)$ asymptotically, i.e.*

$$\lim_{t\to\infty} \int_t^{t+1} \|y(s) - y_{ref}(s)\| ds = 0$$

  for all x_0 and z_0.
- *The above property is robust with respect to perturbations in the parameters (A, B, B_d, C, D)* ***as long as the closed-loop stability is preserved.***

The Internal Model Principle

Theorem (Francis & Wonham, 1970's, LP '10,'14)

*A stabilizing feedback controller solves the robust output regulation problem if and only if it contains "**an internal model**" of the frequencies $\{i\omega_k\}_k$ of the reference signal $y_{\textit{ref}}(t)$.*

The **internal model** for signals with frequencies $\{i\omega_k\}_{k\in\mathcal{I}}$

Every frequency $i\omega_k$ must be an eigenvalue of $\mathcal{G}_1$ with p linearly independent eigenvectors associated to it, i.e.,

$$\dim \mathcal{N}(i\omega_k - \mathcal{G}_1) \geq p$$

Here $p = \dim Y$, the number of outputs.

Controller Design

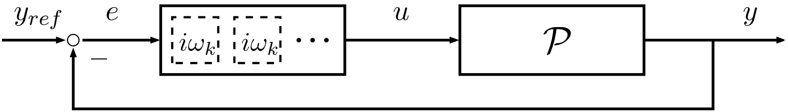
Due to the internal model principle, robust output regulation problem can be solved in two parts:

Step 1° Include a suitable internal model into the controller

Step 2° Use the rest of the controller's parameters to stabilize the closed-loop system.

Internal model has fixed structure, the closed-loop stability can be achieved using one of several alternative ways.

The p-Copy Internal Model Principle



Earlier Work (State Space Techniques)

Robust feedback controllers:

- Hämäläinen-Pohjolainen '96, '00, '10,
- Logemann-Townley '97
- Weiss-Häfele '99,
- Rebarber-Weiss '03,
- Immonen-Pohjolainen '06,
- Boulite-Hadd-Nounou-Nounou '09,
- LP '16,'17.

Regulation without robustness requirement (similar techniques):

- Schumacher '83, Byrnes *et. al.* '00, Natarajan-Gilliam-Weiss '14, Deutcher '15, Xu-Dubljevic '16 etc.

Closed-Loop Stability

To achieve closed-loop stability, two objectives need to be completed:

(1) Stabilization of the possibly unstable plant

(2) Stabilization of the completely unstable internal model

Objective (1) can be approached with observer-based methods.

Hämäläinen-Pohjolainen '10, LP-Pohjolainen '12, '13, LP '15-'17

If the plant is initially stable (or pre-stabilizable), we can concentrate on (2).

$\longrightarrow$ "Simple" controllers whose dynamics contain **only** the internal model.

Simple Controllers

Idea: If the plant is stable (or pre-stabilizable), we can use a **simple** controller where $\mathcal{G}_1$ contains **only** the internal model of $\{i\omega_k\}_k$

To achieve $\dim \mathcal{N}(i\omega_k - \mathcal{G}_1) = \dim Y = p$ for all k, we can choose

$$\mathcal{G}_1 = \operatorname{diag}(i\omega_1 I_{p\times p}, \ldots, i\omega_q I_{p\times p}) \quad \text{or} \quad \mathcal{G}_1 = \operatorname{diag}(i\omega_k I_{p\times p})_{k\in\mathbb{Z}}.$$

on $Z = Y^q = \mathbb{C}^{pq}$ or $Z = \ell^2(Y)$, respectively.

- Ukai-Iwazumi '90, Hämäläinen-Pohjolainen '96, '00, Logemann-Townley '97, Rebarber-Weiss '03 etc.

Closed-Loop Stabilization for Simple Controllers

Goal: Let $Z = Y^q = \mathbb{C}^{pq}$ or $Z = \ell^2(Y)$,

$$\mathcal{G}_1 = \operatorname{diag}(i\omega_1 I_p, \ldots, i\omega_q I_p) \qquad \text{or} \qquad \mathcal{G}_1 = \operatorname{diag}(i\omega_k I_p)_{k\in\mathbb{Z}}$$

and stabilize the closed-loop by choosing $\mathcal{G}_2$ and K in

$$\begin{aligned} \dot{z}(t) &= \mathcal{G}_1 z(t) + \mathcal{G}_2(y_{\mathit{ref}}(t) - y(t)) \qquad z(0) = z_0 \in Z, \\ u(t) &= K z(t) + D_c(y_{\mathit{ref}}(t) - y(t)) \end{aligned}$$

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and stabilize the closed-loop by choosing $\mathcal{G}_2$ and K in

$$\begin{aligned} \dot{z}(t) &= \mathcal{G}_1 z(t) + \varepsilon\, \mathcal{G}_2(y_{ref}(t) - y(t)) \qquad z(0) = z_0 \in Z, \\ u(t) &= K z(t) \end{aligned}$$

Method 1: "Low-gain": Insert an "ε" $\rightarrow$ stability for small $\varepsilon > 0$.

- Powerful for exponentially stable plants, finite sets $\{i\omega_k\}_k$
- Required perturbation argument breaks down for strongly stable plants, nonsmooth $y_{ref}(t)$.

Closed-Loop Stabilization for Simple Controllers

Goal: Let

$$\mathcal{G}_1 = \operatorname{diag}(i\omega_1 I_p, \ldots, i\omega_q I_p) \qquad \text{or} \qquad \mathcal{G}_1 = \operatorname{diag}(i\omega_k I_p)_{k\in\mathbb{Z}}$$

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$$\begin{aligned} \dot{z}(t) &= \mathcal{G}_1 z(t) + \mathcal{G}_2(y_{ref}(t) - y(t)) \qquad z(0) = z_0 \in Z, \\ u(t) &= Kz(t) + D_c(y_{ref}(t) - y(t)), \end{aligned}$$

Method 2: If the plant is "**passive**", choose $(\mathcal{G}_1, \mathcal{G}_2, K)$ passive.

- For infinite-dimensional systems: Rebarber-Weiss '03.
- Very natural approach, no need for small gain parameter.

Passivity: A Very Brief Introduction

Passivity: No internal sources of energy, X Hilbert, $U = Y$,

$$\frac{d}{dt}\|x(t)\|^2 \leq 2\,\mathrm{Re}\langle u(t), y(t)\rangle_Y$$

Examples:

- Many mechanical systems, RLC circuits
- A generates a contraction semigroup, $C = B^*$ with $D = 0$ or $\mathrm{Re}\, D \geq 0$.

Essential property: Under suitable controllability assumptions, can be stabilized with negative output feedback

$$u(t) = -Ky(t), \qquad K > 0.$$

Main Results

Rebarber-Weiss '03: Passive controller for

- exponentially stabilizable systems
- Smooth $y_{ref}(t)$ with finite $\{i\omega_k\}_k$

Here: Extensions: Passive controllers for

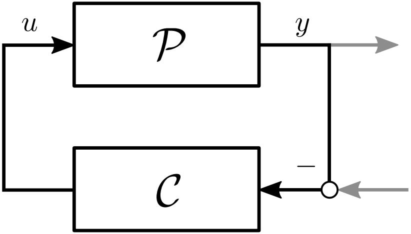
- Strongly stabilizable systems
- Nonsmooth $y_{ref}(t)$, infinite $\{i\omega_k\}_k$
- Detailed analysis of the closed-loop stability and $\|e(t)\| \to 0$

Reference:

LP, "Stability and Robust Regulation of Passive Linear Systems," http://arxiv.org/abs/1706.03224

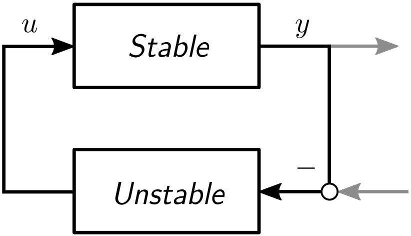
Using Passivity in Stabilization

Main goal: Use passivity to induce closed-loop stability.



A "power-preserving interconnection" $\rightarrow$ Closed-loop semigroup contractive on the Hilbert space $X \times Z$.

Stability of The Closed-Loop System



- Rebarber-Weiss '03: Plant exponentially stabilizable, $\{i\omega_k\}_k$ finite $\Rightarrow$ exponential closed-loop stability

New results: Cases when

- Plant only strongly stabilizable
- $y_{ref}(t)$ nonsmooth with infinite $\{i\omega_k\}_{k\in\mathbb{Z}}$

Stability of The Closed-Loop System

Assume throughout that

- $\mathcal{G}_1 = \operatorname{diag}(i\omega_k I_p)_k$, $\mathcal{G}_2 = K^*$, $(K, \mathcal{G}_1)$ appr. obs., $D_c > 0$
- (A, B, C, D) passive, $\{i\omega_k\}_{k\in\mathbb{Z}}$ infinite
- Denote $\operatorname{Re} T = \frac{1}{2}(T + T^*)$.

Stability of The Closed-Loop System

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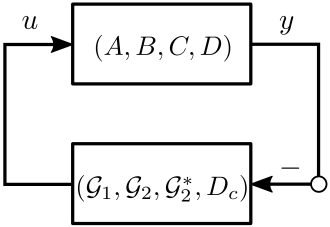
- Denote $\operatorname{Re} T = \frac{1}{2}(T + T^*)$.

Theorem

Assume

- $T(t)$ *is strongly stable,* $i\mathbb{R} \subset \rho(A)$
- $\operatorname{Re} P(i\omega_k) > 0$ *for all* k

Then the closed-loop is strongly stable, $i\mathbb{R} \subset \rho(A_e)$.



- New also for a finite $\{i\omega_k\}_k$.
- finite $\{i\omega_k\}_k \Rightarrow$ robust regulation

[cf. Zhao-Weiss '17]

Stability + IM ⇒ Robust Regulation

For a nonsmooth $y_{ref}(t)$ with an infinite $\{i\omega_k\}_{k\in\mathbb{Z}}$:

Theorem

Assume

- (A,B,C,D) *is strongly stable,* $i\mathbb{R}\subset\rho(A)$
- $\operatorname{Re}P(i\omega_k)>0$ *for all* k

Then the closed-loop is strongly stable. The controller solves the robust output regulation problem for all $y_{ref}(t)=\sum_{k\in\mathbb{Z}}y_r^k e^{i\omega_k t}$ *s.t.*

$$\left(R(i\omega_k,A)BP(i\omega_k)^{-1}y_r^k\right)_k\in\ell^1(X)\qquad\text{and}$$
$$\left(K_k^{-1}P(i\omega_k)^{-1}y_r^k\right)_k\in\ell^2(Y).$$

Here $K_k\in\mathcal{L}(Y)$ are the components of $K=(K_k)_k\in\mathcal{L}(Z,Y)$.

Exponential Closed-Loop Stability

Assume

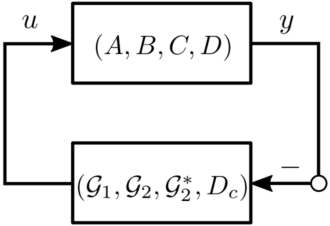
- $\mathcal{G}_1 = \mathrm{diag}(i\omega_k I_p)_k$, $\mathcal{G}_2 = K^*$, $(K, \mathcal{G}_1)$ observable, $D_c > 0$
- (A, B, C, D) passive, $\{i\omega_k\}_{k\in\mathbb{Z}}$ infinite

Theorem

Assume

- $T(t)$ *is exponentially stable*
- $\mathrm{Re}\, P(i\omega_k) \geq d > 0$ *for all* k
- $(K, \mathcal{G}_1)$ *exactly obs.* *($\mathcal{G}_2, K$ unbdd)*

Closed-loop is exponentially stable.



- Restrictive assumptions for both the plant and the controller
- If satisfied $\Rightarrow$ robust regulation

Polynomial (Nonuniform) Closed-Loop Stability

Assume

- $\mathcal{G}_1 = \mathrm{diag}(i\omega_k I_p)_k$, $\mathcal{G}_2 = K^*$, $(K, \mathcal{G}_1)$ observable, $D_c > 0$
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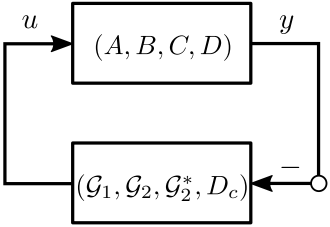

Theorem

Assume K, $\mathcal{G}_2$ bounded,

- $T(t)$ *is exponentially stable*
- $\operatorname{Re} P(i\omega) \geq c|\omega|^{-\beta}$ *for large* $|\omega|$
- $\|K_k^{-1}\|^2 \leq M_K |\omega_k|^\gamma$ *for large* $|k|$

*Closed-loop is **polynomially stable**,*

$$\|T_e(t)x_{e0}\| \leq \frac{M}{t^{1/\alpha}}\|A_e x_{e0}\|,$$

for $x_{e0} \in \mathcal{D}(A_e)$ *where* $\alpha = \beta + \gamma$.

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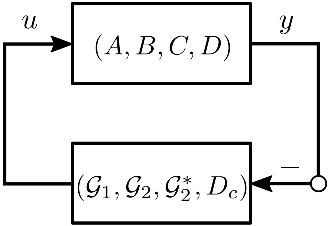

Based on:

Batty-Duyckaerts '08,
Borichev-Tomilov '10.

$\|R(i\omega, A_e)\| \leq O(|\omega|^\alpha)$.

Stability + IM $\Rightarrow$ Robust Regulation

Theorem

Assume $K, \mathcal{G}_2$ bounded

- (A, B, C, D) *is exponentially stable*
- $\operatorname{Re} P(i\omega) \geq c|\omega|^{-\beta}$ *and* $\|K_k^{-1}\|^2 \leq M_K|\omega_k|^\gamma$ *for large* $|\omega|, |k|$

Closed-loop is ***polynomially stable****. The controller solves the regulation problem and for signals* $y_{ref}(t) = \sum_{k\in\mathbb{Z}} y_r^k e^{i\omega_k t}$ *s.t.*

$$\left(\omega_k R(i\omega_k, A)BP(i\omega_k)^{-1}y_r^k\right)_k \in \ell^1 \text{ and } \left(\omega_k K_k^{-1}P(i\omega_k)^{-1}y_r^k\right)_k \in \ell^2$$

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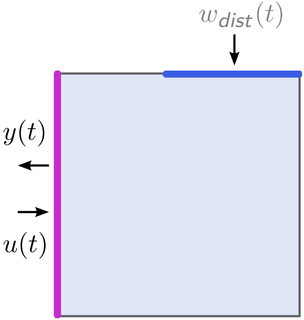
we have

$$\int_t^{t+1} \|y(s) - y_{ref}(s)\| ds = O\left(\frac{1}{t^{1/\alpha}}\right), \qquad \alpha = \beta + \gamma$$

for "suitable" intial states $x_{e0} \in X \times Z$ ($\sim$ classical solutions).

Example

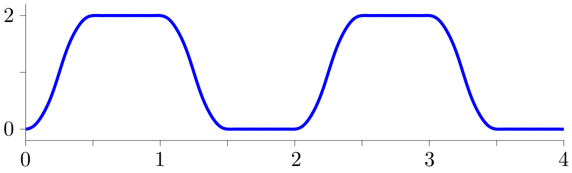
$$x_t(\xi,t) = \Delta x(\xi,t),$$
$$\frac{\partial x}{\partial n}(\xi,t)|_{\Gamma_1} = u(t), \quad \frac{\partial x}{\partial n}(\xi,t)|_{\Gamma_0} = 0$$
$$y(t) = \int_{\Gamma_1} x(\xi,t)d\xi,$$



Defines a regular linear system,

$$|P(i\omega)| = O\left(\frac{1}{\sqrt{|\omega|}}\right) \qquad \text{for large } |\omega|.$$

Reference Signal $y_{ref}(t)$



Consider tracking of a nonsmooth 2-periodic reference signal

$$y_{ref}(t) = \sum_{k\in\mathbb{Z}} \hat{y}_{ref}(k)e^{i\pi kt}$$

where $\omega_k = \pi k$ for $k \in \mathbb{Z}$, and $|\hat{y}_{ref}(k)| = O(|k|^{-3})$.

Robust Controller Construction

Choose $Z = \ell^2(\mathbb{C})$, $\mathcal{G}_1 = \mathrm{diag}(ik\pi)_k$, $\mathcal{G}_2 = K^*$, $D_c > 0$

$$K = \left(\frac{1}{(1+|k|)^{1/2+\varepsilon}} \right)_k \in \mathcal{L}(\ell^2(\mathbb{C}), \mathbb{C}).$$

Then the controller is passive, $\mathcal{G}_2$ and K bounded, $(K, \mathcal{G}_1)$ approximately controllable.

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Then the controller is passive, $\mathcal{G}_2$ and K bounded, $(K, \mathcal{G}_1)$ approximately controllable.

Proposition

The closed-loop is strongly and polynomially stable so that

$$\|T_e(t)x_{e0}\| \le \frac{M}{t^{1/\alpha}} \|A_e x_{e0}\|, \qquad \forall x_{e0} \in \mathcal{D}(A_e),$$

where $\alpha = 3/2 + 2\varepsilon$.

Proof.

$|P(i\omega)| = O(|\omega|^{-1/2})$ and $|K_k|^2 = O(|\omega_k|^{2(1/2+\varepsilon)})$. □

Convergence Rates for the Regulation Error

Proposition

If $0 < \varepsilon < 1/2$, then

$$\int_t^{t+1} \|y(s) - y_{ref}(s)\| ds = O\left(\frac{1}{t^{1/\alpha}}\right), \qquad \alpha = 3/2 + 2\varepsilon$$

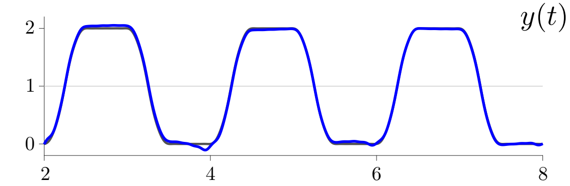
for "suitable" intial states $x_{e0} \in X \times Z$ ($\sim$ classical solutions).

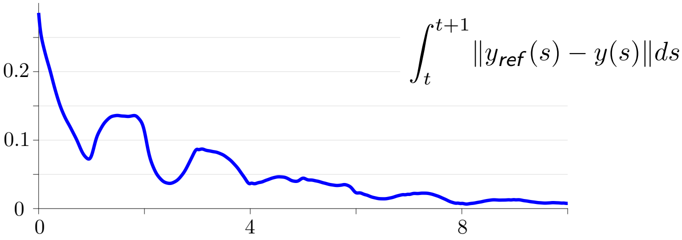
Proof.

The reference signal $y_{ref}(t)$ satisfies

$$\Big(k R(ik\pi, A) B \underbrace{P(ik\pi)^{-1} y_r^k}_{O(|k|^{-5/2})}\Big)_k \in \ell^1 \quad \text{and} \quad \Big(k \underbrace{K_k^{-1} P(ik\pi)^{-1} y_r^k}_{O(|k|^{-2+\varepsilon})}\Big)_k \in \ell^2$$

□





Approximations:

- Finite Differences 20×20 grid,
- $\mathcal{G}_1$ truncated to a 31×31-matrix.

Property:

Proposition

If $\mathcal{G}_2$, K bounded, and $x_0 \in \mathcal{D}(A)$, we can ***choose*** *a "compatible" intial state $z_0 \in Z$ of the controller so that*

$$\int_t^{t+1} \|y(s) - y_{\textit{ref}}(s)\| ds = O\left(\frac{1}{t^{1/\alpha}}\right)$$

is guaranteed.

Conclusions

In this presentation:

- Simple controllers for robust output regulation of passive regular linear systems
- Separate conditions for strong (general), exponential (restrictive), and polynomial stability types
- Analysis of the subexponential convergence rates of the output
- Full closed-loop stability results can also be used in the analysis of coupled PDE-PDE or PDE-ODE systems.

LP, "Stability and Robust Regulation of Passive Linear Systems," http://arxiv.org/abs/1706.03224