A New Controller Structure for Robust Output Regulation

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Main Objectives

Problem

Design controllers for the robust output regulation problem of distributed parameter systems.

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Recent developments by our group:

- A test to determine robustness with respect to a given set of perturbations ("Reduced Order Internal Models").
- The internal model principle for systems with unbounded control and observation (Especially for *regular linear systems*)

Main Objectives

Problem

Design controllers for the robust output regulation problem of distributed parameter systems.

In this presentation:

- A new “complementary” controller structure for robust output regulation.

Consider a plant

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \qquad x(0) = x_0 \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

where

- $u(t) \in \mathbb{C}^m$ is the control input
- $y(t) \in \mathbb{C}^p$ is the measured output.

Consider a plant

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad x(0) = x_0$$
$$y(t) = Cx(t) + Du(t)$$

General assumptions on B and C:

- $B \in \mathcal{L}(\mathbb{C}^m, X_{-1})$ and $C \in \mathcal{L}(X_1, \mathbb{C}^p)$ are admissible
- (A, B, C, D) is regular, i.e., for one/all $\lambda \in \rho(A)$

$$P(\lambda) = C_\Lambda R(\lambda, A_{-1})B + D$$

is well-defined and $P(\cdot) \in H^\infty(\mathbb{C}_\beta^+)$ for some $\beta \in \mathbb{R}$.

The Control Problem

Problem (Robust Output Regulation)

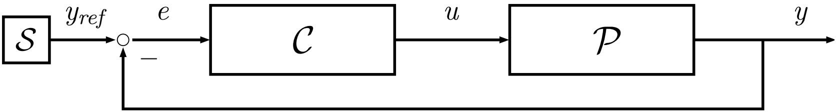
Choose a control law in such a way that

- *The closed-loop semigroup is stable.*
- *The output $y(t)$ tracks a given reference signal $y_{ref}(t)$ asymptotically, i.e.*

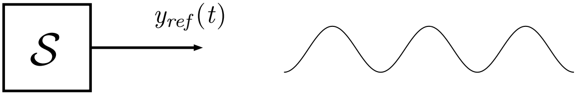
$$\lim_{t\to\infty} \|y(t) - y_{ref}(t)\| = 0$$

- *The above property is robust with respect to small perturbations in the operators (A, B, C, D) of the plant.*

The Exosystem and the Control Scheme



The Reference Signals and the Exosystem



$y_{ref}(t)$ are of the form

$$y_{ref}(t) = \sum_{k=1}^{q} y_k e^{i\omega_k t}, \qquad y_k \in \mathbb{C}^p.$$

It is customary to interpret $y_{ref}(t)$ as an output of an exosystem

$$\dot{v}(t) = Sv(t), \qquad v_0 \in \mathbb{C}^q$$
$$y_{ref}(t) = Fv(t)$$

where $S = \operatorname{diag}(i\omega_1, i\omega_2, \dots, i\omega_q)$ and $F \in \mathbb{C}^{p \times q}$.

The Dynamic Error Feedback Controller

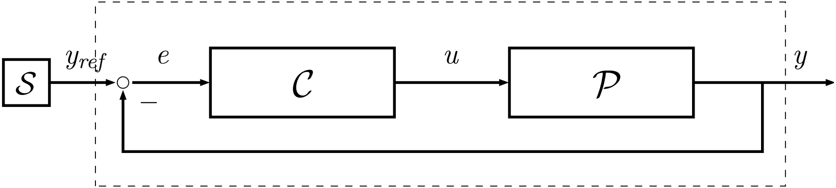
We consider an error feedback controller $(\mathcal{G}_1, \mathcal{G}_2, K)$ of the form

$$\begin{aligned} \dot{z}(t) &= \mathcal{G}_1 z(t) + \mathcal{G}_2(y(t) - y_{ref}(t)), \qquad z(0) = z_0 \in Z \\ u(t) &= Kz(t), \end{aligned}$$

where $K \in \mathcal{L}(Z_1, U)$ is admissible and $\mathcal{G}_2 \in \mathcal{L}(Y, Z)$.

Feedback controllers are known to be essential in achieving robustness.

The Closed-Loop System



Closed-loop system

Property: The closed-loop operator

$$A_e = \begin{pmatrix} A_{-1} & BK \\ \mathcal{G}_2 C_\Lambda & \mathcal{G}_1 + \mathcal{G}_2 DK \end{pmatrix}$$

$$\mathcal{D}(A_e) = \left\{ (x, z) \in \mathcal{D}(C_\Lambda) \times \mathcal{D}(K) \;\middle|\; A_{-1}x + BKz \in X \right\}$$

generates a strongly continuous semigroup $T_e(t)$ on $X \times Z$.

The Internal Model Principle

The first new (and old) result:

Theorem (Francis & Wonham, 1970's, LP & SP 2010–2014)

If the controller is such that A_e generates an exponentially stable semigroup, then it solves the robust output regulation problem if and only if it contains p copies of the dynamics of the exosystem.

Here $p = \dim Y$, the number of outputs.

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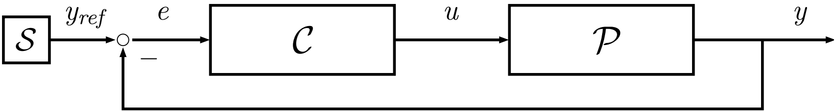
Here $p = \dim Y$, the number of outputs.

The p-copy for an exosystem with $S = \operatorname{diag}(i\omega_1, \ldots, i\omega_q)$:

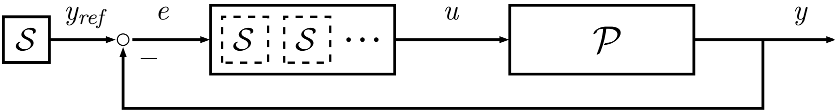
Any eigenvalue $i\omega_k$ of S must be an eigenvalue of $\mathcal{G}_1$ with p linearly independent eigenvectors associated to it, i.e.,

$$\dim \mathcal{N}(i\omega_k - \mathcal{G}_1) \geq p.$$

Feedback Controller



The p-Copy Internal Model Principle



IMP for Unbounded B and C Needs Novel Tactics...

Theorem (A Key Step & More)

A stabilizing controller $(\mathcal{G}_1, \mathcal{G}_2, K)$ is robust with respect to given perturbations $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) \in \mathcal{O}$ if and only if the equations

$$\tilde{P}(i\omega_k)Kz^k = -Fe_k$$

$$(i\omega_k - \mathcal{G}_1)z^k = 0$$

have a solution $z^k \in \mathcal{D}(\mathcal{G}_1)$ for all $k \in \{1, \ldots, q\}$.

Here: e_k is an Euclidean basis vector, F is the output operator of the exosystem, $\mathcal{G}_1$ is the system operator and K the output operator of the controller.

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have a solution $z^k \in \mathcal{D}(\mathcal{G}_1)$ *for all* $k \in \{1, \dots, q\}$*.*

In addition to proving the IMP, can be used in studying robustness of the controller with respect to *individual perturbations*, leads to "Reduced Order Internal Models", TAC'2013.

Controller Construction

The CDC'14 paper, main contribution:

Proposition

We introduce a new controller structure for robust output regulation with an error feedback controller of the form

$$\dot{z}(t) = \mathcal{G}_1 z(t) + \mathcal{G}_2(y(t) - y_{ref}(t)), \qquad z(0) = z_0 \in Z$$
$$u(t) = Kz(t),$$

For simplicity, for bounded $B \in \mathcal{L}(U, X)$ and $C \in \mathcal{L}(X, Y)$.

Background: Structure by Hämäläinen & Pohjolainen

The triangular structure (based on Francis & Wonham '74)

$$\dot{z}(t) = \begin{pmatrix} G_1 & 0 \\ R_1 & R_2 \end{pmatrix} z(t) + \begin{pmatrix} G_2 \\ R_3 \end{pmatrix} (y(t) - y_{ref}(t))$$
$$u(t) = \begin{pmatrix} K_1 & K_2 \end{pmatrix} z(t)$$

- (G_1, G_2) contain the internal model of the exosystem
- Other operators are used in stabilization (observer)

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Pros and cons:

(+) Natural if the IM is defined via $\mathcal{G}$-*conditions* (F&W'74).

(−) Difficult for the p-copy version of the internal model

(−) Does not work for reduced order internal models.

In This Paper: A New Structure

A complementary triangular structure,

$$\dot{z}(t) = \begin{pmatrix} G_1 & R_1 \\ 0 & R_2 \end{pmatrix} z(t) + \begin{pmatrix} G_2 \\ R_3 \end{pmatrix} (y(t) - y_{ref}(t))$$
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- (G_1, K_1) contain the internal model of the exosystem
- Other operators are used in stabilization (observer)

Pros and cons:

(+) Natural if the internal model is defined using the p-copy

(+) Can be used for reduced order internal models!

(−) Difficult for $\mathcal{G}$-conditions

Comparison of the Two Structures

The Old One: $K = (K_1,\ K_2)$

$$\mathcal{G}_1 = \begin{pmatrix} G_1 & 0 \\ R_1 & R_2 \end{pmatrix}, \quad \mathcal{G}_2 = \begin{pmatrix} G_2 \\ R_3 \end{pmatrix}$$

- (G_1, G_2) is the IM
- $(R_1, R_2, R_3, K_1, K_2)$ used in stabilization

(+) Good for $\mathcal{G}$-conds

The New One: $K = (K_1,\ K_2)$

$$\mathcal{G}_1 = \begin{pmatrix} G_1 & R_1 \\ 0 & R_2 \end{pmatrix}, \quad \mathcal{G}_2 = \begin{pmatrix} G_2 \\ R_3 \end{pmatrix}$$

- (K_1, G_1) is the IM
- $(R_1, R_2, R_3, G_2, K_2)$ used in stabilization

(+) Good for p-copy and ROIM

Comparison: Choices of R_1, R_2, R_3

The Old One:

$$\mathcal{G}_1 = \begin{pmatrix} G_1 & 0 \\ (B+LD)K_1 & A+BK_2+L(C+DK_2) \end{pmatrix}, \quad \mathcal{G}_2 = \begin{pmatrix} G_2 \\ -L \end{pmatrix},$$

The New One:

$$\mathcal{G}_1 = \begin{pmatrix} G_1 & G_2(C+DK_2) \\ 0 & A+BK_2+L(C+DK_2) \end{pmatrix}, \qquad \mathcal{G}_2 = \begin{pmatrix} G_2 \\ L \end{pmatrix},$$

References

LP – *Designing controllers with reduced order internal models*, TAC (tech. note), 2015.

LP & S. Pohjolainen – *The Internal Model Principle for Systems with Unbounded Control and Observation*, SIAM J. Control Optim., to appear, 2015.

LP & S. Pohjolainen – *Reduced order internal models in robust output regulation*, Transactions on Automatic Control, 2013.

Conclusions

In this presentation.

- Robust output regulation and the internal model for plants with unbounded B and C.
- A new triangular controller structure for robust output regulation.