Robust Output Regulation for Infinite-Dimensional Systems

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Main Objective

Problem

Generalize the Internal Model Principle of robust output regulation to distributed parameter systems and nonsmooth reference signals.

1. Introduction
2. The Robust Output Regulation Problem
3. Generalization of the Internal Model Principle
4. Further Results on Robustness
5. Conclusions

Consider a plant

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0$$
$$y(t) = Cx(t) + Du(t)$$

where $u(t)$ is the control input and $y(t)$ the measured output.

The plant is an infinite-dimensional system on a Banach space X. Covers PDEs, systems with delays, transport equations, infinite platoons etc.

The Control Problem

Problem (Robust Output Regulation)

Choose a control law in such a way that

- *The output* $y(t)$ *tracks a given reference signal* $y_{ref}(t)$ *asymptotically, i.e.*

$$\lim_{t\to\infty} \|y(t) - y_{ref}(t)\| = 0$$

- *The above property is robust with respect to small uncertainties and changes in the parameters of the plant.*

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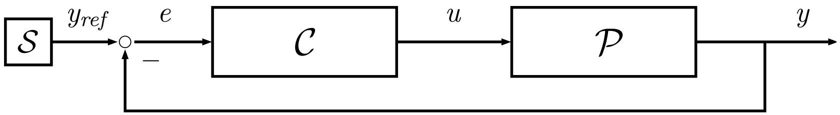
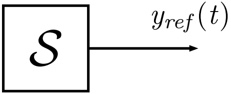
Origins:

For finite-dimensional linear systems, Francis & Wonham, Davison in the 1970's.

The Exosystem and the Control Scheme



Classes of Periodic Nonsmooth Reference Signals



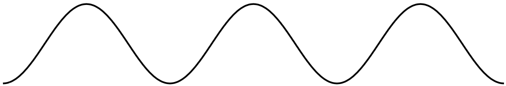


Figure: Examples of generated reference signals

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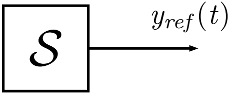

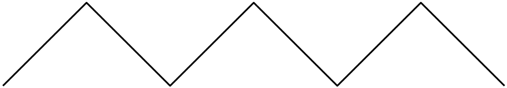

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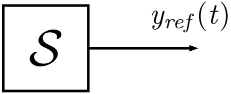

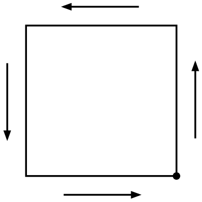

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The Exosystems, An Overview

The infinite-dimensional exosystem

$$\begin{aligned} \dot{v}(t) &= Sv(t), \qquad v_0 \in W \\ y_{ref}(t) &= Fv(t) \end{aligned}$$

on a Hilbert space W, with S unbounded block-diagonal operator.

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Signals:

$$y_{ref}(t) = t^n y_n(t) + \cdots + ty_1(t) + y_0(t)$$

where $y_j(\cdot)$ are periodic functions.

The Internal Model Principle

Theorem (Francis & Wonham, 1970's)

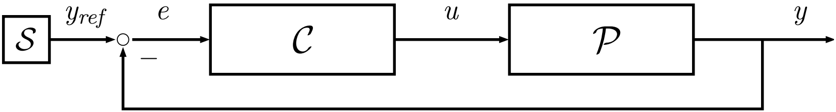
A stabilizing feedback controller solves the robust output regulation problem if and only if it contains p copies of the dynamics of the signal generator.

Here $p = \dim Y$, the number of outputs.

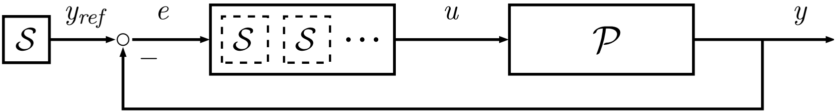
The "contains p copies of the dynamics" means (roughly) that

> *for any Jordan block in the signal generator there must be p Jordan blocks of equal or greater size in the controller (associated to the same eigenvalue).*

Feedback Controller



The p-Copy Internal Model Principle



Earlier Work on Generalizing the IMP

- B. A. Francis & W. M. Wonham – The finite-dimensional Internal Model Principle, 1970's
- M. K. P. Bhat – Partial extension for distributed parameter systems, 1976
- E. Immonen – Partial extension for nonsmooth reference signals, 2006
- Y. Yamamoto – In the frequency domain, 1988.

Main Result

LP & S. Pohjolainen, SIAM J. Control Optim. (2010):

Theorem

Generalization of the Internal Model Principle by Francis & Wonham for distributed parameter systems and nonsmooth reference signals.

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i.e. for every Jordan block of dimension n in the exosystem, the system operator of the controller must have p Jordan chains of lengths $\geq n$.

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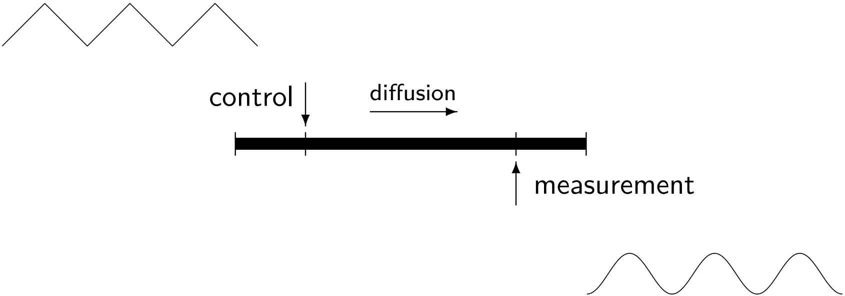
Due to infinite-dimensionality and nonsmooth signals: Additional conditions relating

- Behavior of the system's transfer function at infinity on $i\mathbb{R}$ (Note, not holomorphic at ∞ for DPS).
- The smoothness properties of the reference signals.

Example: Heating of a metal bar



Remarks on the Theorem

Remark

The proof of the Internal Model Principle is largely based on requiring robustness with respect to perturbations to the output operators of the exosystem.

Allowing such perturbations is often unnecessary (in particular, if reference signals are known accurately).

Robustness w.r.t. a Restricted Class of Perturbations

Problem

If we are only interested in robustness with respect to a specific class of perturbations, we can then ask

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how big an internal model do we need?

i.e., how many times must the dynamics of the exosystem be copied in the controller.

Main Result

A *full* internal model is necessary for robustness with respect to all small perturbations in any one of the operators.

Theorem

If the control law is robust with respect to all small rank one perturbations in any one of the operators A, B, C, or D of the plant, then the controller necessarily incorporates a p-copy internal model of the exosystem.

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LP & S. Pohjolainen - *Reduced Order Internal Models in Robust Output Regulation*, Submitted.

Example: A MIMO Wave Equation

Set-point regulation ($y_{ref}(t) \equiv y_r \in \mathbb{C}^p$ or $y_r \in Y$ Banach space) of a system

$$\frac{\partial^2 w}{\partial t^2}(z,t) - \alpha \frac{\partial w}{\partial t}(z,t) = \frac{\partial^2 w}{\partial z^2}(z,t) + Bu(t)$$

$$y(t) = Cw(\cdot,t).$$

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We can build a 1-dimensional controller that is robust with respect to all sufficiently small perturbations in α.

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Conclusions

In this presentation.

- Internal Model Principle for distributed parameter systems with infinite-dimensional exosystems.
- A more detailed look into perturbations and robustness properties.