

Title: State and parameter estimation of hyperbolic PDEs coupled with nonlinear ODEs

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Description of the Postdoc

I. Context

Hyperbolic Partial Differential Equations (PDEs) model transport-dominated phenomena characterized by finite propagation speed. These equations arise in a wide range of applications, including hydraulic network modeling [5, Ch. 8], road traffic dynamics [13], gas pipeline networks, and drilling operations [1]. In this context, **state and parameter estimation** is essential for real-time monitoring and output-feedback control. Recent advances in this field have largely relied on the **backstepping method** [9], which has enabled the design of observers with exponential or even finite-time convergence guarantees. This approach has become a cornerstone for state observer design in coupled hyperbolic systems [4, 12]. More recently, observer design has been extended to **interconnected systems**, such as PDE–ODE cascades and networks of coupled PDEs [11]. However, most existing designs focus on **linear systems**. Since actuator dynamics may exhibit nonlinearities (such as saturations) it becomes necessary to develop new methods for the observability of cascaded systems involving nonlinear ODEs and PDEs. We believe a promising direction lies in **combining the backstepping approach with another powerful tool for the observability of nonlinear ODEs: the Kazantzis–Kravaris/Luenberger (KKL) observer**. Originally developed in [8] and [3], this observer builds on the foundational methodology introduced by D. Luenberger for linear systems [10]. A recent exposition of the theory is provided in [7]. It is also known to be efficient for parameter estimation, under persistency of excitation conditions [2]. While all these works focus on finite-dimensional systems, the recent preprint [6] has shown how to adapt the KKL observer in the specific framework of a nonlinear ODEs cascaded with a 1D heat equation. This breakthrough motivates the objective of this project.

II. Scientific objective and methodology

Postdoc goal: The general objective of this postdoctoral project is to design **state and parameter observers** for hyperbolic systems coupled with nonlinear finite-dimensional dynamics.

We propose an original methodology that integrates concepts from both **backstepping** and the **Kazantzis–Kravaris/Luenberger (KKL)** methodology. The observer will be designed by employing a **KKL observer** for the nonlinear ODE and a **backstepping observer** for the linear PDE. The motivation for this work stems from the observation of **structural similarities** between backstepping observers and KKL observers in the context of linear dynamics. Indeed, both of them rely on embedding the original system into a stable linear filter of the measured output, and proving that the embedding is injective. Our approach builds upon the methodology presented in [7, Section 4], which addresses observer design for cascaded systems comprising a nonlinear ODE followed by a linear ODE in normal form. This methodology will be adapted to handle **unknown parameters** and to develop functional KKL-backstepping-based observers. Implementing this approach raises several scientific challenges, which will be addressed during the postdoctoral research:

- **Adjusting the KKL methodology to infinite-dimensional systems:** The KKL mapping must be **injective** to ensure the convergence of the observer. This property will be established using an appropriate **differential observability** or **backward distinguishability** condition.
- **Enhancing state observers with a parameter estimation component:** We will demonstrate that certain parameters can be reconstructed if the system is **persistently excited** by an input.

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We will build upon the parameter estimation methodology based on the KKL observer developed in [2] for finite-dimensional systems.

- **Numerical implementation of the proposed observers:** Practical validation through numerical simulations will be essential to assess the effectiveness and robustness of the designed observers. In particular, the numerical left-inversion of the backstepping and KKL embedding will be challenging due to their infinite-dimensional nature.

III. Desired profile

This postdoctoral position requires a strong background in control systems and applied mathematics. The ideal candidate should hold a Ph.D. in control theory with experience in observer design for nonlinear systems or infinite-dimensional systems. Expertise in parameter estimation would be valued. Additionally, the candidate should have a proven track record of publishing high-quality research articles.

IV. Application

To apply, write an email with your CV and a summary of your research activities to Lucas Brivadis and Jean Auriol: `firstname.lastname@centralesupelec.fr`.

V. Advisors' Biographies

Lucas Brivadis (supervision load: 50%).

He received in 2018 his engineering degree from École Centrale de Lyon and his master's degree in applied mathematics from Université Lyon 1. He defended his PhD in 2021 at LAGEPP, Université Lyon 1. From 2021 to 2022, he was a postdoctoral researcher at L2S, (CNRS, CentraleSupélec, Université Paris-Saclay). Since 2022, he is a CNRS researcher (chargé de recherche) at L2S. His research interests include observer design for infinite-dimensional and nonlinear systems.

Jean AURIOL (supervision load: 50%). He received his Master's degree in civil engineering in 2015 (major: applied maths) from MINES ParisTech, part of PSL Research University, and in 2018 his PhD degree in control theory and applied mathematics from the same university (Centre Automatique et Systèmes). From 2018 to 2019, he was postdoctoral researcher at the Department of Petroleum Engineering, University of Calgary, Canada, where he was working on implementing backstepping control laws to attenuate mechanical vibrations in drilling systems. Since December 2019, he has been an associate researcher (Chargé de Recherche) at CNRS, Université Paris-Saclay, CentraleSupélec, L2S. His research interests include robust control of distributed parameter systems and interconnected systems.

VI. References

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