Decentralized Observer Design for Virtual Decomposition Control

Jukka-Pekka Humaloja[1], Janne Koivumäki[2], Lassi Paunonen[1], Jouni Mattila[2]

***Abstract*—In this paper, we incorporate velocity observer design into the virtual decomposition control (VDC) strategy of an n-DoF open chain robotic manipulator. Descending from the VDC strategy, the proposed design is based on decomposing the n-DoF manipulator into subsystems, i.e., rigid links and joints, for which the decentralized controller-observer implementation can be done locally. Similar to VDC, the combined controller-observer design is passivity-based, and we show that it achieves semiglobal exponential convergence of the tracking error. The convergence analysis is carried out using Lyapunov functions based on the observer and controller error dynamics. The proposed design is demonstrated in a simulation study of a 2-DoF open chain robotic manipulator in the vertical plane.**

***Index Terms*—decentralized controller-observer design, velocity observer, nonlinear control, virtual decomposition control**

I. Introduction

The virtual decomposition control (VDC) approach [1], [2] is a nonlinear model-based control method that is developed for controlling complex systems, and it has been demonstrated to be very effective especially in robotic control [3]–[7]. The fundamental idea of VDC is that the system can be virtually decomposed into *modular subsystems* (such as rigid links and joints), allowing a decentralized control that can be designed locally at the subsystem level and that guarantees stability by fully taking into account the dynamic interactions among adjacent subsystems. The VDC methodology is introduced in greater detail in Section II-B.

The existing VDC literature requires that the position and velocity states of the system are measurable for the control design. While position measurements can be done accurately, the instruments for measuring rotation speed, e.g., tachometers, are known to be often contaminated with noise. Velocity data can naturally be obtained by numerical differentiation of the position sensor data but there is no theoretical justification for this method [8], [9]. Due to these challenges, control of n-DoF robotic manipulators without velocity data has been extensively studied, e.g., in [8], [10]–[15], see also the survey [9], where the actuator dynamics have been neglected. Similar research for robots with, e.g., hydraulic actuators has been done in [16], [17]. Our approach to the proposed controller-observer design is inspired by the passivity-based design in [8] and the subsystem-based VDC approach [1].

In this paper, we design a control law for an n-DoF open chain robotic manipulator (see Fig. 1) in such a way that the position trajectories $q_i(t)$ of the n joints follow given *desired trajectories* $q_{id}(t)$. Velocity data is not available for the control design, due to which we design a velocity observer based on position (measurement) and torque (input) data. We note that the manipulator in Fig. 1 is in a planar joint configuration for the sake of graphical simplicity; the system kinematics and dynamics are provided in the general 6-DoF matrix/vector form instead of the scalar presentation and the joint orientations may be arbitrary.

The main contribution of the paper is incorporating an observer design into the VDC methodology, which yields a novel decentralized controller-observer design for robotic manipulators. In comparison to the existing literature [8]–[17] where the designs are based on the dynamics of the whole manipulator, in the proposed decentralized design the control and observer gains are proportional to the individual link/joint dynamics. Thus, in the proposed design the gain conditions remain unaltered even if the complexity of the system (number of DoFs) increases, which is not the case for the existing designs where the gain conditions depend on the whole system dynamics. Moreover, the proposed design is highly modular in the sense that if parts were replaced in or added to the manipulator, the controller-observer design needs to be reimplemented only for the new parts while the other parts remain intact. Our main result, semiglobal exponential convergence of the proposed design, is presented in Theorem VI.3 in Section VI. Thereafter, Remark VI.5 discusses possible extensions of the design and addresses arbitrary joint configurations for the n-DoF manipulator. Semiglobality of the achieved convergence originates from the requirement of the link velocities being bounded, albeit they may be arbitrarily large. We note that a similar approach has been taken, e.g., in [8].

The proposed controller-observer design is based on the VDC design principles in the sense that the controller and observer are designed for the links and joints, i.e., the virtual subsystems, individually, and the stability analysis of the error dynamics can be carried out locally at the subsystem level in terms of *virtual stability* (see Section II-B). The idea is to construct Lyapunov functions for the subsystems based on the error dynamics, using which the error dynamics can be shown to be exponentially stable. The approach is an integral part of VDC in controller stability analysis, and in this paper we

[1]J.-P. Humaloja and L. Paunonen are with Mathematics, Faculty of Information Technology and Communication Sciences, Tampere University, Finland. e-mail: firstname.surname@tuni.fi

[2]J. Koivumäki and J. Mattila are with Automation Technology and Mechanical Engineering, Faculty of Engineering and Natural Sciences, Tampere University, Finland. e-mail: firstname.surname@tuni.fi

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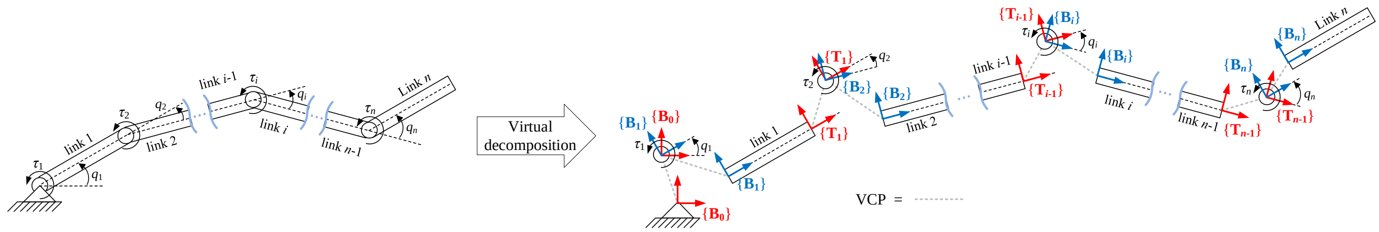

Fig. 1. The n-DoF open chain robotic manipulator and its virtual decomposition.

extend the design and analysis to account for the observer error dynamics as well. It should be noted that due to the nonlinear dynamics of the system, the separation principle cannot be utilized in stability analysis but the controller and observer convergences must be shown simultaneously.

The paper is organized as follows. In Section II, we present preliminaries concerning link dynamics, stability analysis and the VDC methodology. In Section III, we present the kinematics and dynamics of the system model. In Section IV, we present the decentralized joint and link velocity observer designs which are incorporated into the VDC design in Section V. Exponential stability of the controller and observer error dynamics is shown in Section VI. In Section VII, the proposed design is demonstrated by a numerical simulation on a 2-DoF robot in the vertical plane. Finally, the paper is concluded in Section VIII.

II. Mathematical Preliminaries

A. Dynamics of a Rigid Body

Consider an orthogonal, three-dimensional coordinate system $\{\mathbf{A}\}$ (frame $\{\mathbf{A}\}$) attached to a rigid body. Let ${}^{\mathbf{A}}\mathbf{v} \in \mathbb{R}^3$ and ${}^{\mathbf{A}}\omega \in \mathbb{R}^3$ be the linear and angular velocity vectors, respectively, of frame $\{\mathbf{A}\}$, expressed in frame $\{\mathbf{A}\}$ (see [1, Sect. 2.5] for expressing velocities and forces in body frames). To facilitate the transformations of velocities among different frames, the linear/angular velocity vector of frame $\{\mathbf{A}\}$ can be written as

$$ {}^{\mathbf{A}}V := \begin{bmatrix} {}^{\mathbf{A}}\mathbf{v} \\ {}^{\mathbf{A}}\omega \end{bmatrix} \in \mathbb{R}^6. \tag{1} $$

In a similar manner, let ${}^{\mathbf{A}}\mathbf{p} \in \mathbb{R}^3$ and ${}^{\mathbf{A}}\varphi \in \mathbb{R}^3$ be the linear and angular position vectors, respectively, of frame $\{\mathbf{A}\}$ and define ${}^{\mathbf{A}}P := \begin{bmatrix} {}^{\mathbf{A}}\mathbf{p} \\ {}^{\mathbf{A}}\varphi \end{bmatrix} \in \mathbb{R}^6$, so that $\frac{d}{dt}({}^{\mathbf{A}}P) = {}^{\mathbf{A}}V$.

Let ${}^{\mathbf{A}}\mathbf{f} \in \mathbb{R}^3$ and ${}^{\mathbf{A}}\mathbf{m} \in \mathbb{R}^3$ be the force and moment vectors applied to the origin of frame $\{\mathbf{A}\}$, expressed in frame $\{\mathbf{A}\}$. Similar to (1), the force/moment vector in frame $\{\mathbf{A}\}$ can be written as

$$ {}^{\mathbf{A}}F := \begin{bmatrix} {}^{\mathbf{A}}\mathbf{f} \\ {}^{\mathbf{A}}\mathbf{m} \end{bmatrix} \in \mathbb{R}^6. \tag{2} $$

Consider two given frames, denoted as $\{\mathbf{A}\}$ and $\{\mathbf{B}\}$, fixed to a common rigid body. The following relations hold:

$$ {}^{\mathbf{B}}V = {}^{\mathbf{A}}\mathbf{U}_{\mathbf{B}}^{T}\,{}^{\mathbf{A}}V \tag{3a} $$
$$ {}^{\mathbf{A}}F = {}^{\mathbf{A}}\mathbf{U}_{\mathbf{B}}\,{}^{\mathbf{B}}F, \tag{3b} $$

where ${}^{\mathbf{A}}\mathbf{U}_{\mathbf{B}} \in \mathbb{R}^{6\times 6}$ denotes a force/moment transformation matrix that transforms the force/moment vector measured and expressed in frame $\{\mathbf{B}\}$ to the same force/moment vector measured and expressed in frame $\{\mathbf{A}\}$ (see [1, Sect. 2.5.3] for details).

Let frame $\{\mathbf{A}\}$ be fixed to a rigid body. The rigid body dynamics, expressed in frame $\{\mathbf{A}\}$, can be written as

$$ \mathbf{M_A}\frac{d}{dt}({}^{\mathbf{A}}V) + \mathbf{C_A}({}^{\mathbf{A}}\omega){}^{\mathbf{A}}V + \mathbf{G_A} = {}^{\mathbf{A}}F^{*} \tag{4} $$

where ${}^{\mathbf{A}}F^{*} \in \mathbb{R}^6$ is the net force/moment vector of the rigid body expressed in frame $\{\mathbf{A}\}$ and $\mathbf{M_A} \in \mathbb{R}^{6\times 6}$, $\mathbf{C_A}({}^{\mathbf{A}}\omega) \in \mathbb{R}^{6\times 6}$ and $\mathbf{G_A} \in \mathbb{R}^6$ are the mass matrix, the Coriolis/centrifugal matrix and the gravity vector, respectively (see [1, Sect. 2.6.2] for the detailed expressions). We note that by the structure of matrix $\mathbf{C_A}(\cdot)$, it has the following properties

$$ \mathbf{C_A}({}^{\mathbf{A}}\omega_1)^T = -\mathbf{C_A}({}^{\mathbf{A}}\omega_1) \tag{5a} $$
$$ \mathbf{C_A}(\alpha_1{}^{\mathbf{A}}\omega_1 + \alpha_2{}^{\mathbf{A}}\omega_2) = \alpha_1\mathbf{C_A}({}^{\mathbf{A}}\omega_1) + \alpha_2\mathbf{C_A}({}^{\mathbf{A}}\omega_2) \tag{5b} $$
$$ \|\mathbf{C_A}({}^{\mathbf{A}}\omega_1)\| \le M_{c,\mathbf{A}}\|{}^{\mathbf{A}}\omega_1\| \tag{5c} $$

for some $M_{c,\mathbf{A}} > 0$ and for all $\alpha_1, \alpha_2 > 0$ and ${}^{\mathbf{A}}\omega_1, {}^{\mathbf{A}}\omega_2 \in \mathbb{R}^3$.

B. Virtual Decomposition Control

Virtual decomposition control (VDC) is a control design method where the original system is decomposed into subsystems by placing conceptual *virtual cutting points* (VCP) [1, Def. 2.13]. Every such cutting point forms a virtual cutting surface on the rigid body, where three-dimensional force vectors and three-dimensional moment vectors can be exerted from one part to another. Fig. 1 displays a virtual decomposition of an n-DoF robot and the virtual cutting points.

Adjacent subsystems resulting from a virtual decomposition have dynamic interactions with each other. These interactions are uniquely defined by scalar terms called *virtual power flows* (VPFs) [1, Def. 2.16]. With respect to frame $\{\mathbf{A}\}$, the virtual power flow is given by

$$ p_{\mathbf{A}} = ({}^{\mathbf{A}}V_r - {}^{\mathbf{A}}V)^T({}^{\mathbf{A}}F_r - {}^{\mathbf{A}}F) \tag{6} $$

where ${}^{\mathbf{A}}V_r \in \mathbb{R}^6$ and ${}^{\mathbf{A}}F_r \in \mathbb{R}^6$ represent the required vectors of ${}^{\mathbf{A}}V \in \mathbb{R}^6$ and ${}^{\mathbf{A}}F \in \mathbb{R}^6$, respectively, that will be presented in Section V.

The VPFs are closely related to *virtual stability* [1, Def. 2.17] which is the key concept of VDC. Virtual stability is a tool for analyzing the stability of the system on a subsystem level, where the subsystems do not need to be stable but using VPFs to represent dynamic interactions among adjacent subsystems. Motivated by [1, Def. 2.17], we define virtual stability as follows:

Definition II.1: Consider a subsystem i with dynamics $\dot{x}_i = g_i(t,x_i)$ where g_i is piecewise continuous in t and locally Lipschitz in x_i. The subsystem i is called *virtually stable* if there exists a continuously differentiable function $\nu_i(t,x_i)$ such that

$$\alpha_{i,1}\|x_i\|^2 \le \nu_i(t,x_i) \le \alpha_{i,2}\|x_i\|^2 \quad (7a)$$

$$\dot{\nu}_i(t,x_i) \le -\alpha_{i,3}\|x_i\|^2 + p(t,x_i)_{\mathbf{A}_{i+1}} - p(t,x_i)_{\mathbf{A}_{i-1}} \quad (7b)$$

for all $t \ge 0$ and some $\alpha_{i,1}, \alpha_{i,2}, \alpha_{i,3} > 0$, where $p_{\mathbf{A}_{i+1}}$ and $p_{\mathbf{A}_{i-1}}$ are VPFs with respect to frames $\{\mathbf{A}_{i+1}\}$ and $\{\mathbf{A}_{i-1}\}$, respectively, adjacent to subsystem i.

Note that without the VPFs, Definition II.1 would coincide with Lyapunov criteria for exponential stability as in [18, Thm. 4.10]. The virtually stable subsystems will in fact result in exponential stability of the entire system, as the VPFs in (7b) will cancel out when we take the sum of the functions ν_i over all the subsystems. We will prove the stability of the entire system in Theorem VI.3. The approach is the same as in [1, Thm. 2.1], even though the stability arguments here are different.

Remark II.2: The VDC design is decentralized and local in the sense that changing the control (or dynamics) of a subsystem does not affect the control equations of the rest of the system as long as the VPFs among adjacent subsystems cancel out. We also note that the general concept of virtual stability in [1, Def. 2.17] allows several VPFs between the subsystems. That is, the concept is not restricted to open chain systems but the restriction is made here merely for simplicity. The general formulation of VDC is given in [1, Sect. 4].

III. The System Model

Consider the robot with n links as in Fig. 1 with the given virtual decomposition. For the sake of generality, we will formulate the kinematics and dynamics of the system in the matrix/vector form in $\mathbb{R}^6$. For more detailed consideration of the kinematics and dynamics, see [1, Chap. 3] where the consideration is done for a 2-DoF robot.

A. Kinematics

Let the system base frame $\{\mathbf{B}_0\}$ have zero velocity, i.e., ${}^{\mathbf{B}_0}V = \mathbf{0}$. Then, using the notation of Section II-A, the kinematics of an arbitrary joint i can be written as

$${}^{\mathbf{B}_i}V = \mathbf{z}_\tau \dot{q}_i + {}^{\mathbf{B}_{i-1}}\mathbf{U}_{\mathbf{B}_i}^T\, {}^{\mathbf{B}_{i-1}}V, \quad i \in \{1,2,\ldots,n\}, \quad (8)$$

where $\mathbf{z}_\tau = [0\ 0\ 0\ 0\ 0\ 1]^T$, $\dot{q}_i$ is the angular velocity of joint i and ${}^{\mathbf{B}_{i-1}}\mathbf{U}_{\mathbf{B}_i}$ is as in (3). Moreover, as the following relation holds for transforming the velocity vectors in link i:

$${}^{\mathbf{T}_i}V = {}^{\mathbf{B}_i}\mathbf{U}_{\mathbf{T}_i}^T\, {}^{\mathbf{B}_i}V, \quad i \in \{1,2,\ldots,n\}, \quad (9)$$

the kinematics of joint i can alternatively be written based on the link velocities as

$${}^{\mathbf{B}_i}V = \mathbf{z}_\tau \dot{q}_i + {}^{\mathbf{T}_{i-1}}U_{\mathbf{B}_i}^T\, {}^{\mathbf{T}_{i-1}}V, \quad i \in \{2,3,\ldots,n\}. \quad (10)$$

B. Single Link Dynamics in Cartesian Space

As in (4), the motion dynamics of an arbitrary rigid link i is expressed in frame $\{\mathbf{B}_i\}$ by

$$\mathbf{M}_{\mathbf{B}_i}\frac{d}{dt}({}^{\mathbf{B}_i}V) + \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\omega)^{\mathbf{B}_i}V + \mathbf{G}_{\mathbf{B}_i} = {}^{\mathbf{B}_i}F^*, \quad i \in \{1,2,\ldots,n\}. \quad (11)$$

Furthermore, the resultant forces/moments of link i can be expressed as

$${}^{\mathbf{B}_i}F = {}^{\mathbf{B}_i}F^* + {}^{\mathbf{B}_i}\mathbf{U}_{\mathbf{T}_i}\, {}^{\mathbf{T}_i}F, \quad i \in \{1,2,\ldots,n\}, \quad (12)$$

where ${}^{\mathbf{T}_n}F = \mathbf{0}$ as we assume that no external force/moment is imposed on the origin of the frame $\{\mathbf{T}_n\}$. Moreover, the force/moment vector in frame $\{\mathbf{B}_0\}$ can be written as

$${}^{\mathbf{B}_0}F = {}^{\mathbf{B}_0}U_{B_1}\, {}^{\mathbf{B}_1}F. \quad (13)$$

C. Single Joint Dynamics in Joint Space

The actuation torque of an arbitrary joint i can be obtained from (12) as

$$\tau_{ai} = \mathbf{z}_\tau^T\, {}^{\mathbf{B}_i}F, \quad i \in \{1,2,\ldots,n\}. \quad (14)$$

Then, similarly to [1, (3.51)], joint i torque τ_i (torque input) can be written as

$$\tau_i = I_{m,i}\ddot{q}_i + f_{c,i}(\dot{q}_i) + \tau_{ai}, \quad i \in \{1,2,\ldots,n\} \quad (15)$$

where $I_{m,i}$ is the joint moment of inertia and $f_{c,i}$ is a Coulomb friction function model. The friction model is assumed to be increasing, globally Lipschitz continuous and antisymmetric, e.g., Coulomb-viscous model (see [19, Sect. 2.3]) Note that by monotonicity, the friction function model satisfies

$$-(x_1 - x_2)(f_{c,i}(x_1) - f_{c,i}(x_2)) \le 0. \quad (16)$$

We note that the monotonicity assumption could be lifted if the first time derivatives of the functions $f_{c,i}$ are bounded, so that more advanced friction models (see [19, Sect. 3]) could be incorporated as well.

IV. Observer Design

In this section, we will consider velocity observers for arbitrary link i and joint i motivated by the passivity-based observer design of [8, Sect. II.B]. For the design, we need to have position and torque data available. The final observer designs must be done simultaneously with the control designs (see [8, Sect. II.C]), which we will do in Section V, where the following auxiliary analysis will be utilized. We make the following standing assumption:

Assumption IV.1: Joint torque data τ_i and position data q_i are available for the observer design for all $i \in \{1,2,\ldots,n\}$.

A. Observer for Link i

Consider an observer system of the form

$${}^{\mathbf{B}_i}\dot{\hat{P}} = {}^{\mathbf{B}_i}Z - \mathbf{M}_{\mathbf{B}_i}^{-1}\mathbf{L}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{P} - {}^{\mathbf{B}_i}P) \quad (17a)$$

$$\mathbf{M}_{\mathbf{B}_i}\, {}^{\mathbf{B}_i}\dot{Z} = {}^{\mathbf{B}_i}F^* - \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{\omega})^{\mathbf{B}_i}\hat{V} - \mathbf{G}_{\mathbf{B}_i} \quad (17b)$$

where $\mathbf{L}_{\mathbf{B}_i} > 0$ is an error feedback gain matrix, $[{}^{\mathbf{B}_i}\hat{P}\quad {}^{\mathbf{B}_i}Z]^T$ is the observer state and ${}^{\mathbf{B}_i}\hat{V} = {}^{\mathbf{B}_i}\dot{\hat{P}}$ is the observed velocity.

Note that the second line of the observer simply copies the link dynamics (11).

Subtracting (11) from (17b), we obtain error dynamics

$$\begin{aligned}\mathbf{M}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\dot{\hat{V}} - {}^{\mathbf{B}_i}\dot{V}) = \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\omega){}^{\mathbf{B}_i}V - \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{\omega}){}^{\mathbf{B}_i}\hat{V} \\ - \mathbf{L}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V). \qquad (18)\end{aligned}$$

If we define a quadratic function $\nu_{\mathbf{B}_i,obs}$ as

$$\nu_{\mathbf{B}_i,obs} := \frac{1}{2}({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V)^T \mathbf{M}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V), \qquad (19)$$

then

$$\begin{aligned}\dot{\nu}_{\mathbf{B}_i,obs} = ({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V)^T[\mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\omega){}^{\mathbf{B}_i}V - \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{\omega}){}^{\mathbf{B}_i}\hat{V}] \\ - ({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V)^T \mathbf{L}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V). \qquad (20)\end{aligned}$$

The term associated with the Coriolis/centrifugal forces can be written as

$$\begin{aligned}&\mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\omega){}^{\mathbf{B}_i}V - \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{\omega}){}^{\mathbf{B}_i}\hat{V} \qquad (21)\\ &= \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\omega){}^{\mathbf{B}_i}V - \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{\omega}){}^{\mathbf{B}_i}V - \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{\omega})({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V),\end{aligned}$$

so by using (5a)–(5b) we obtain

$$\begin{aligned}&({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V)^T[\mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\omega){}^{\mathbf{B}_i}V - \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{\omega}){}^{\mathbf{B}_i}\hat{V}]\\ &= ({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V)^T[\mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\omega){}^{\mathbf{B}_i}V - \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{\omega}){}^{\mathbf{B}_i}V] \qquad (22)\\ &= ({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V)^T \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\omega - {}^{\mathbf{B}_i}\hat{\omega}){}^{\mathbf{B}_i}V.\end{aligned}$$

Let us now assume that the velocity vector ${}^{\mathbf{B}_i}V$ is bounded, i.e., $\sup_{t>0}\|{}^{\mathbf{B}_i}V\| = M_{v,i} < \infty$. Continuing from (22) and using the relative boundedness (5c) of $\mathbf{C}_{\mathbf{B}_i}(\cdot)$, we obtain

$$\begin{aligned}&\|({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V)^T \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\omega - {}^{\mathbf{B}_i}\hat{\omega}){}^{\mathbf{B}_i}V\|\\ &\le \|{}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V\| M_{c,i} M_{v,i} \|{}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V\|. \qquad (23)\end{aligned}$$

Utilizing the preceding identities and estimates in (20), we finally obtain

$$\dot{\nu}_{\mathbf{B}_i,obs} \le -({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V)^T(\mathbf{L}_{\mathbf{B}_i} - M_{c,i}M_{v,i}I_{6\times6})({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V) \quad (24)$$

which can be made negative by choosing $\mathbf{L}_{\mathbf{B}_i} > M_{c,i}M_{v,i}I_{6\times6}$. We will fix the choice for $\mathbf{L}_{\mathbf{B}_i}$ later when designing the combined controller-observer in Section V.

B. Observer for Joint i

Similarly as in the case of link i, we design a velocity observer for an arbitrary joint i, namely

$$\dot{\hat{q}}_i = z_i - L_i(\hat{q}_i - q_i) \qquad (25a)$$

$$I_m \dot{z}_i = \tau_i - \tau_{ai} - f_{c,i}(\dot{\hat{q}}_i) - \ell_i(\hat{q}_i - q_i) \qquad (25b)$$

where $[\hat{q}_i \quad z_i]^T$ is the observer state, $L_i, \ell_i > 0$ are gain parameters and $\dot{\hat{q}}_i$ is the observed (angular) velocity. Unlike the observer for link i, the proposed observer also contains a position error feedback term which is added to achieve position convergence in addition to velocity convergence.

Before computing the error dynamics, we set $L_i = \ell_i + I_{m,i}^{-1}$. Now, subtracting (15) from (25b), we obtain error dynamics

$$\begin{aligned}I_{m,i}(\ddot{\hat{q}}_i - \ddot{q}_i) = -[f_{c,i}(\dot{\hat{q}}_i) - f_{c,i}(\dot{q}_i)]\\ - (I_{m,i}\ell_i + 1)(\dot{\hat{q}}_i - \dot{q}_i) - \ell_i(\hat{q}_i - q_i), \qquad (26)\end{aligned}$$

which by defining a new variable $s_i := (\dot{\hat{q}}_i - \dot{q}_i) + \ell_i(\hat{q}_i - q_i)$ can be equivalently written as $I_{m,i}\dot{s}_i = -[f_{c,i}(\dot{\hat{q}}_i) - f_{c,i}(\dot{q}_i)] - s_i$. Let us now define a quadratic function $\nu_{i,obs}$ as

$$\nu_{i,obs} := \frac{I_{m,i}}{2}(\dot{\hat{q}}_i - \dot{q}_i)^2 + \frac{\ell_i}{2}(\hat{q}_i - q_i)^2 + \frac{I_{m,i}}{2}s_i^2. \qquad (27)$$

Then, denoting the Lipschitz constant of $f_{c,i}$ by $m_{c,i}$, we obtain

$$\begin{aligned}\dot{\nu}_{i,obs} &= -(\dot{\hat{q}}_i - \dot{q}_i)(f_{c,i}(\dot{\hat{q}}_i) - f_{c,i}(\dot{q}_i)) - I_{m,i}L_i(\dot{\hat{q}}_i - \dot{q}_i)^2\\ &\quad - \ell_i(\dot{\hat{q}}_i - \dot{q}_i)(\hat{q}_i - q_i) + \ell_i(\dot{\hat{q}}_i - \dot{q}_i)(\hat{q}_i - q_i)\\ &\quad - s_i^2 - s_i(f_{c,i}(\dot{\hat{q}}_i) - f_{c,i}(\dot{q}_i))\\ &\le -I_{m,i}L_i(\dot{\hat{q}}_i - \dot{q}_i)^2 - s_i^2 - s_i(f_{c,i}(\dot{\hat{q}}_i) - f_{c,i}(\dot{q}_i))\\ &\le -I_{m,i}L_i(\dot{\hat{q}}_i - \dot{q}_i)^2 - s_i^2 + \frac{s_i^2}{2} + \frac{1}{2}(f_{c,i}(\dot{\hat{q}}_i) - f_{c,i}(\dot{q}_i))^2\\ &\le -I_{m,i}L_i(\dot{\hat{q}}_i - \dot{q}_i)^2 - s_i^2 + \frac{s_i^2}{2} + \frac{m_{c,i}^2}{2}(\dot{\hat{q}}_i - \dot{q}_i)^2\\ &= -\left(I_{m,i}L_i - \frac{m_{c,i}^2}{2}\right)(\dot{\hat{q}}_i - \dot{q}_i)^2 - \frac{1}{2}s_i^2, \qquad (28)\end{aligned}$$

the right-hand-side of which is negative for all $L_i > \frac{1}{2}m_{c,i}^2 I_{m,i}^{-1}$. We will fix the choice of L_i (or rather ℓ_i) as part of the joint controller-observer design in the next section.

Remark IV.2: Note that $\ell_i(\hat{q}_i - q_i) = s_i - (\dot{\hat{q}}_i - \dot{q}_i)$ so that $0 \le \ell_i(\hat{q}_i - q_i)^2 \le 2\ell_i^{-1}s_i^2 + 2\ell_i^{-1}(\dot{\hat{q}}_i - \dot{q}_i)^2$.

V. Control with Observers

In order to achieve position control for the system, let us introduce the concept of *required joint i velocity* as $\dot{q}_{ir} = \dot{q}_{id} + \lambda_i(q_{id} - q_i)$, where q_{id} is the traditionally used desired position trajectory (see [20] for q_{id}) for joint i and $\lambda_i > 0$ is a control parameter [1, Sect. 3.3.6]. However, because we later need to be able to realize $\ddot{q}_{ir}$ for the control, we redefine the required velocity according to [8, Sect. III.A] as

$$\dot{q}_{ir} := \dot{q}_{id} + \lambda_i(q_{id} - \hat{q}_i), \qquad (29)$$

where we use the observed position $\hat{q}_i$ in place of q_i.

A. Control of Link i

In line with (8), the required linear/angular velocity vector at link i base frame $\{\mathbf{B}_i\}$ can be obtained as

$${}^{\mathbf{B}_i}V_r = \mathbf{z}_\tau \dot{q}_{ir} + {}^{\mathbf{B}_{i-1}}\mathbf{U}_{\mathbf{B}_i}^T\, {}^{\mathbf{B}_{i-1}}V_r \quad i \in \{1, 2, \ldots, n\}, \qquad (30)$$

and, in line with (9), the following relation holds for transforming the required linear/angular velocity vectors in link i:

$${}^{\mathbf{T}_i}V_r = {}^{\mathbf{B}_i}\mathbf{U}_{\mathbf{T}_i}^T\, {}^{\mathbf{B}_i}V_r, \quad i \in \{1, 2, \ldots, n\} \qquad (31)$$

with ${}^{\mathbf{B}_0}V_r = \mathbf{0}$ in (30) and (31). Then, in view of (11) and using (30), the required net force/moment vector for link i can be written as

$$\begin{aligned}{}^{\mathbf{B}_i}F_r^* = \mathbf{M}_{\mathbf{B}_i}\frac{d}{dt}({}^{\mathbf{B}_i}V_r) + \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{\omega}){}^{\mathbf{B}_i}V_r + \mathbf{G}_{\mathbf{B}_i}\\ + \mathbf{K}_{\mathbf{B}_i}({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}\hat{V}), \quad i \in \{1, 2, \ldots, n\} \qquad (32)\end{aligned}$$

where $\mathbf{K}_{\mathbf{B}_i} > 0$ is a velocity gain matrix. Note that we need to use the observed velocities in the matrix $\mathbf{C}_{\mathbf{B}_i}$ and in the feedback term. Finally, the required force/moment vector can be written by reusing (12) as

$$ {}^{\mathbf{B}_i}F_r = {}^{\mathbf{B}_i}F_r^* + {}^{\mathbf{B}_i}\mathbf{U}_{\mathbf{T}_i}{}^{\mathbf{T}_i}F_r, \quad i \in \{1,2,\dots,n\}. \tag{33} $$

Let us define a quadratic function $\nu_{\mathbf{B}_{i,ctrl}}$ for link i as

$$ \nu_{\mathbf{B}_{i,ctrl}} := \frac{1}{2}({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)^T \mathbf{M}_{\mathbf{B}_i}({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V). \tag{34} $$

Motivated by the discussion in [8, Sect. II.C], we define a quadratic function $\nu_{\mathbf{B}_i}$ for link i as

$$ \nu_{\mathbf{B}_i} := \nu_{\mathbf{B}_i,ctrl} + \nu_{\mathbf{B}_i,obs}, \tag{35} $$

where $\nu_{\mathbf{B}_1,obs}$ is given in (19). The following lemma provides an auxiliary result that will be utilized in Section VI when proving exponential convergences of the observation and control for the whole n-DoF system.

Lemma V.1: If the controller gain $\mathbf{K}_{\mathbf{B}_i}$ in (32) is chosen such that $\mathbf{K}_{\mathbf{B}_i} > I_{6\times6}$ and the observer gain $\mathbf{L}_{\mathbf{B}_i}$ in (17) is chosen such that $\mathbf{L}_{\mathbf{B}_i} > M_{c,i}M_{v,i}\left(1+\frac{1}{2}M_{c,i}M_{v,i}\right)I_{6\times6} + \frac{1}{2}\mathbf{K}_{\mathbf{B}_i}$, then there exist some $M_{i,1}, M_{i,2} > 0$ such that $\nu_{\mathbf{B}_i}$ in (35) satisfies

$$ \begin{aligned} \dot{\nu}_{\mathbf{B}_i} \leq &-({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)^T M_{i,1}({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V) \\ &-({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V)^T M_{i,2}({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V) + p_{\mathbf{B}_i} - p_{\mathbf{T}_i} \end{aligned} \tag{36} $$

for all $i \in \{1,2,\dots,n\}$.

Proof: See Appendix A. ■

B. Control of Joint i

We already saw in the previous section that, in line with (8), the required joint and link velocities are related by (30). Alternatively, in line with (10) we can write

$$ {}^{\mathbf{B}_i}V_r = \mathbf{z}_\tau \dot{q}_{ir} + {}^{\mathbf{T}_{i-1}}U_{\mathbf{B}_i}^T{}^{\mathbf{T}_{i-1}}V_r, \quad i \in \{2,3,\dots,n\}. \tag{37} $$

Then, in view of (14)–(15), the control law for joint i can be written as

$$ \tau_{air} = \mathbf{z}_\tau^T{}^{\mathbf{B}_i}F_r \tag{38a} $$
$$ \tau_i = I_{m,i}\ddot{q}_{ir} + f_{c,i}(\dot{q}_{ir}) + \tau_{air} + k_i(\dot{q}_{ir} - \hat{\dot{q}}_i) \tag{38b} $$
$$ {}^{\mathbf{B}_0}F_r = {}^{\mathbf{B}_0}U_{\mathbf{B}_1}{}^{\mathbf{B}_1}F_r \tag{38c} $$

where $k_i > 0$ is a velocity feedback gain. Similar to Lemma V.1, we have the following auxiliary result:

Lemma V.2: For all $k_i > 0$ in (38), if the observer gains ℓ_i and L_i in (25) are chosen such that $2I_{m,i}L_i > \max\{2, m_{c,i}^2 + k_i\}$ and $\ell_i = L_i - I_{m,i}^{-1} > 0$, then there exists some $m_i > 0$ such that the quadratic function

$$ \nu_{ai} := \frac{I_{m,i}}{2}(\dot{q}_{ir} - \dot{q}_i)^2 + \frac{I_{m,i}}{2}(\hat{\dot{q}}_i - \dot{q}_i)^2 + \frac{\ell_i}{2}(\hat{q}_i - q_i)^2 + \frac{I_{m,i}}{2}s_i^2, \tag{39} $$

where $s_i = (\hat{\dot{q}}_i + \dot{q}_i) + \ell_i(\hat{q}_i - q_i)$, satisfies

$$ \dot{\nu}_{ai} \leq -\frac{1}{2}k_i(\dot{q}_{ir} - \dot{q}_i)^2 - m_i(\hat{\dot{q}}_i - \dot{q}_i)^2 - \frac{1}{2}s_i^2 + p_{\mathbf{A}_{i-1}} - p_{\mathbf{B}_i} \tag{40} $$

for all $i \in \{1,2,\dots,n\}$, where we denote $p_{\mathbf{A}_0} = p_{\mathbf{B}_0}$ and $p_{\mathbf{A}_i} = p_{\mathbf{T}_i}$ for $i \in \{1,2,\dots,n-1\}$.

Proof: See Appendix B. ■

VI. Stability of the Entire System

In order to prove that the proposed controller-observer design achieves tracking of the desired trajectories, recall that the base frame $\{\mathbf{B}_0\}$ has zero velocity and that no external forces[1] are imposed on the origin of frame $\{\mathbf{T}_n\}$. Thus, ${}^{\mathbf{B}_0}V = {}^{\mathbf{B}_0}V_r = \mathbf{0}$ and ${}^{\mathbf{T}_n}F = {}^{\mathbf{T}_n}F_r = \mathbf{0}$ so that $p_{\mathbf{B}_0} = p_{\mathbf{T}_n} = 0$. Now we can construct a Lyapunov function for the whole system by summing over the functions $\nu_{\mathbf{B}_i}$ and ν_{ai}, as the virtual power flows appearing in the time derivatives of the functions will cancel out in the summation.

In Section IV-A we assumed that the link velocities are bounded, but—as shown in the proof of Theorem VI.3 below—this can be guaranteed by assuming the desired joint velocities $\dot{q}_{id}$ to be bounded and by restricting to a suitable set of initial conditions. Thus, we make the following assumption:

Assumption VI.1: There exist some $M_{d,i}, M'_{d,i} > 0$ such that $|q_{id}| \leq M_{d,i}$ and $|\dot{q}_{id}| \leq M'_{d,i}$ for all $t \geq 0$ and $i \in \{1,2,\dots,n\}$. Moreover, for simplicity we assume that the gain matrices $\mathbf{L}_{\mathbf{B}_i}, \mathbf{K}_{\mathbf{B}_i}$ are constant and diagonal, that is:

Assumption VI.2: $\mathbf{L}_{\mathbf{B}_i} = L_{\mathbf{B}_i}I_{6\times6}$ and $\mathbf{K}_{\mathbf{B}_i} = K_{\mathbf{B}_i}I_{6\times6}$ for all $i \in \{1,2,\dots,n\}$, where $L_{\mathbf{B}_i}, K_{\mathbf{B}_i} > 0$.

We will now show that the combined observer-control law converges exponentially for all initial conditions satisfying

$$ \|\mathbf{x}(0)\| < \min_{i\in\{1,2,\dots,n\}} \left\{ \sqrt{\frac{\alpha_m}{\alpha_M}} \frac{\frac{\sqrt{1+2L_{\mathbf{B}_i}-K_{\mathbf{B}_i}}-1}{M_{c,i}} - \sum_{k=1}^{i} M_U^k M'_{d,k}}{1 + \sum_{k=1}^{i} M_U^k \frac{16\lambda_k}{4\lambda_k - \alpha_M^{-1}}} \right\} \tag{41} $$

with $\hat{q}_i(0) = q_{id}(0)$ and $4\lambda_i > \alpha_M^{-1}$ for all $i \in \{1,2,\dots,n\}$, where

$$ \begin{aligned} \mathbf{x}^T &= \left[({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)^T, ({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V)^T, \dot{q}_{ir} - \dot{q}_i, \hat{\dot{q}}_i - \dot{q}_i, s_i\right]_{i=1}^n \\ \alpha_m &= \min\{\min(\sigma(\mathbf{M}_{\mathbf{B}_i})), I_{m,i}\}_{i=1}^n \\ \alpha_M &= \max\left\{\max(\sigma(\mathbf{M}_{\mathbf{B}_i})), I_{m,i} + \ell_i^{-1}\right\}_{i=1}^n, \\ M_U &= \max\left\{1, \|{}^{\mathbf{B}_0}\mathbf{U}_{\mathbf{B}_1}\|, \|{}^{\mathbf{B}_1}\mathbf{U}_{\mathbf{B}_2}\|, \dots, \|{}^{\mathbf{B}_{n-1}}\mathbf{U}_{\mathbf{B}_n}\|\right\}, \end{aligned} $$

where $\sigma(\cdot)$ denotes the set of eigenvalues. Note that the gains $\mathbf{L}_{\mathbf{B}_i}, \mathbf{K}_{\mathbf{B}_i}$ and λ_i can be assigned independently for all $i \in \{1,2,\dots,n\}$, and that the region characterized by (41) can be made arbitrarily large by increasing the gains $\mathbf{L}_{\mathbf{B}_i}$ (while keeping the other parameters fixed). Thus, the region of attraction is *semiglobal*.

Theorem VI.3: Under the standing assumptions and for all initial conditions satisfying (41), the combined observer-control law described in (17), (25), (29)–(33) and (37)–(38) with the gains chosen according to Lemmas V.1 and V.2, the tracking errors $q_{id} - q_i$ decay exponentially to zero for all $i \in \{1,2,\dots,n\}$.

Proof: By Lemmas V.1 and V.2 and using $p_{\mathbf{B}_0} = p_{\mathbf{T}_n} = 0$, the quadratic function

$$ \nu := \sum_{i=1}^{n}(\nu_{\mathbf{B}_i} + \nu_{ai}), \tag{42} $$

[1] Constrained motion control (i.e., contacts with the environment) can be addressed in VDC with a VPF appearing between the manipulator and the environment (see [1], [6], [21]), but this topic is outside the scope of the present study.

with $v_{\mathbf{B}_i}$ and v_{ai} given in (35) and (39), respectively, satisfies

$$\begin{aligned}\dot{v} &= \sum_{i=1}^{n}(\dot{v}_{\mathbf{B}_i}+\dot{v}_{ai})\\ &\leq \sum_{i=1}^{n}\Big[-({}^{\mathbf{B}_i}V_r-{}^{\mathbf{B}_i}V)^T M_{i,1}({}^{\mathbf{B}_i}V_r-{}^{\mathbf{B}_i}V)+p_{\mathbf{B}_i}\\ &\qquad -({}^{\mathbf{B}_i}\hat{V}-{}^{\mathbf{B}_i}V)^T M_{i,2}({}^{\mathbf{B}_i}\hat{V}-{}^{\mathbf{B}_i}V)-p_{\mathbf{T}_i}\\ &\qquad -k_i(\dot{q}_{ir}-\dot{q}_i)^2-m_i(\dot{\hat{q}}_i-\dot{q}_i)^2-\frac{1}{2}s_i^2\\ &\qquad +p_{\mathbf{T}_i}-p_{\mathbf{B}_i}\Big]\\ &= \sum_{i=1}^{n}\Big[-({}^{\mathbf{B}_i}V_r-{}^{\mathbf{B}_i}V)^T M_{i,1}({}^{\mathbf{B}_i}V_r-{}^{\mathbf{B}_i}V)\\ &\qquad -({}^{\mathbf{B}_i}\hat{V}-{}^{\mathbf{B}_i}V)^T M_{i,2}({}^{\mathbf{B}_i}\hat{V}-{}^{\mathbf{B}_i}V)\\ &\qquad -k_i(\dot{q}_{ir}-\dot{q}_i)^2-m_i(\dot{\hat{q}}_i-\dot{q}_i)^2-\frac{1}{2}s_i^2\Big].\end{aligned} \tag{43}$$

Using Remark IV.2 and [18, Thm. 4.10], we have that $\frac{\alpha_m}{2}\|\mathbf{x}\|^2 \leq v \leq \frac{\alpha_M}{2}\|\mathbf{x}\|^2$. Moreover, by (43) and Lemmas V.1 and V.2 we have that $\dot{v} \leq -\alpha_p\|\mathbf{x}\|^2$ for $\alpha_p = \min\left\{\min(\sigma(M_{i,1})),\min(\sigma(M_{i,2})),k_i,m_i,\frac{1}{2}\right\}_{i=1}^{n} > 0$. However, positivity of α_p requires that especially the gain condition in Lemma V.1 holds, which under Assumption VI.2 reduces to

$$L_{\mathbf{B}_i} > M_{c,i}\|{}^{\mathbf{B}_i}V\|(1+\frac{1}{2}M_{c,i}\|{}^{\mathbf{B}_i}V\|)+\frac{1}{2}K_{\mathbf{B}_i} \tag{44}$$

for all $i\in\{1,2,\ldots,n\}$ and $t\geq 0$. We will show that this is achieved for all initial conditions satisfying (41).

Let $i\in\{1,2,\ldots,n\}$ be arbitrary. In order to estimate $\|{}^{\mathbf{B}_i}V\|$, we begin by subtracting $\dot{\hat{q}}_i$ from both sides of (29). Rearranging terms yields linear dynamics $\dot{q}_{id}-\dot{\hat{q}}_i = -\lambda_i(q_{id}-\hat{q}_i)+\dot{q}_{ir}-\dot{\hat{q}}_i$, where by the above Lyapunov analysis $|\dot{q}_{ir}-\dot{\hat{q}}_i| \leq |\dot{q}_{ir}-\dot{q}_i|+|\dot{\hat{q}}_i-\dot{q}_i| \leq 2\sqrt{\frac{\alpha_M}{\alpha_m}}\|\mathbf{x}(0)\|\exp(-\frac{\alpha_p}{2\alpha_M}t)$ for all $t\geq 0$. Thus, by the variation of parameters formula we obtain

$$q_{id}-\hat{q}_i = (q_{id}(0)-\hat{q}_i(0))e^{-\lambda_i t}+\int_0^t e^{-\lambda_i(t-s)}(\dot{q}_{ir}-\dot{\hat{q}}_i)ds \tag{45}$$

and since by assumption $q_{id}(0)=\hat{q}_i(0)$, we can estimate

$$\sup_{t\geq 0}|q_{id}-\hat{q}_i| \leq \frac{4}{\lambda_i-\frac{\alpha_p}{2\alpha_M}}\sqrt{\frac{\alpha_M}{\alpha_m}}\|\mathbf{x}(0)\|$$

where $\lambda_i > \frac{\alpha_p}{2\alpha_M}$ by assumption. Thus, by (29) we obtain

$$\sup_{t\geq 0}|\dot{q}_{ir}| \leq \sup_{t\geq 0}|\dot{q}_{id}|+\frac{4\lambda_i}{\lambda_i-\frac{\alpha_p}{2\alpha_M}}\sqrt{\frac{\alpha_M}{\alpha_m}}\|\mathbf{x}(0)\|$$

and consequently by (30) and Assumption VI.1 we obtain

$$\begin{aligned}\sup_{t\geq 0}\|{}^{\mathbf{B}_i}V_r\| &\leq \sum_{k=1}^{i} M_U^k \sup_{t\geq 0}|\dot{q}_{kr}|\\ &\leq \sum_{k=1}^{i} M_U^k M'_{d,k}+\sqrt{\frac{\alpha_M}{\alpha_m}}\|\mathbf{x}(0)\|\sum_{k=1}^{i} M_U^k\frac{16\lambda_k}{4\lambda_k-\alpha_M^{-1}},\end{aligned}$$

where we also used $\alpha_p \leq \frac{1}{2}$. Finally, we have that

$$\begin{aligned}\sup_{t\geq 0}\|{}^{\mathbf{B}_i}V\| &\leq \sup_{t\geq 0}\|{}^{\mathbf{B}_i}V_r\|+\sup_{t\geq 0}\|{}^{\mathbf{B}_i}V_r-{}^{\mathbf{B}_i}V\|\\ &\leq \sup_{t\geq 0}\|{}^{\mathbf{B}_i}V_r\|+\sqrt{\frac{\alpha_M}{\alpha_m}}\|\mathbf{x}(0)\|\\ &< \frac{\sqrt{1+2L_{\mathbf{B}_i}-K_{\mathbf{B}_i}}-1}{M_{c,i}}\end{aligned}$$

for all $\mathbf{x}(0)$ satisfying (41), i.e., (44) holds.

Now that we have shown that the error dynamics is exponentially stable with a given region of attraction, this implies by linearity that $\hat{q}_i-q_i = \ell_i^{-1}(s_i-(\dot{\hat{q}}_i-\dot{q}_i))$ decay exponentially to zero for all $i\in\{1,2,\ldots,n\}$. Finally, based on (45) we have that $q_{id}-q_i = (q_{id}-\hat{q}_i)+(\hat{q}_i-q_i)$ decay exponentially to zero for all $i\in\{1,2,\ldots,n\}$, which concludes the proof. ■

Remark VI.4: As ${}^{\mathbf{B}_i}\hat{V}-{}^{\mathbf{B}_i}V\to 0$ for all $i\in\{1,2,\ldots,n\}$ by Theorem VI.3, it follows that ${}^{\mathbf{B}_i}\hat{P}$ in (17) converges up to a constant from ${}^{\mathbf{B}_i}P$ and hence remains bounded by Theorem VI.3 and Assumption VI.1.

Remark VI.5: Note that as long as position and total torque data is available, the observers are in fact independent of the coordinate frames as there are no observer-based virtual power flows between neighboring frames. That is, as long as we can make the observers stable at the subsystem level, the proposed observer design could potentially be incorporated into more general VDC designs [1, Sect. 4] as well. Note also that the present design is not limited to planar joint configuration as the joint orientations can be altered freely by changing the direction vector $\mathbf{z}_\tau$.

VII. Numerical Simulation of a 2-DoF Robot

For an example, consider a robot as in Fig. 1 with two links of length $l_1=l_2=1$. Similar to [22, Sect. 2.1], both links are modeled as point masses $m_1=m_2=1$ at the distal ends so that the rotational inertia for both links is $I_1=I_2=m_2l_2^2=1$.

In line with (4), the link dynamics are given by

$$\mathbf{M}_{\mathbf{B}_i}\frac{d}{dt}({}^{\mathbf{B}_i}V)+\mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\omega){}^{\mathbf{B}_i}V+\mathbf{G}_{\mathbf{B}_i} = {}^{\mathbf{B}_i}F^* \tag{46}$$

where [1, Sect. 3.4]

$$\mathbf{M}_{\mathbf{B}_i} = \begin{bmatrix} m_i & 0 & 0\\ 0 & m_i & m_il_i\\ 0 & m_il_i & I_i+m_il_i^2\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&1\\0&1&2\end{bmatrix}, \tag{47}$$

$$\mathbf{C}_{\mathbf{B}_i}(\omega) = \begin{bmatrix}0 & -m_i & -m_il_i\\ m_i & 0 & 0\\ m_il_i & 0 & 0\end{bmatrix}\omega = \begin{bmatrix}0&-1&-1\\1&0&0\\1&0&0\end{bmatrix}\omega, \tag{48}$$

and $\mathbf{G}_{\mathbf{B}_i} = [0,0,m_ig_il_i\cos(q_i)]^T = [0,0,9.81\times\cos(q_i)]^T$ for $i\in\{1,2\}$. The net forces ${}^{\mathbf{B}_i}F^*$ are obtained based on (12), where the transformation matrices are given in [1, Sect 3.3.2]. The link dynamics comprise two linear components and one angular component. For further details on the 2-DoF example, see [1, Sect. 3]. The joint dynamics are given in line with (15) by $I_{m,i}\ddot{q}_i = \tau_i-\tau_{ai}-f_{c,i}(\dot{q}_i)$, where $I_{m,i}=0.1$ and $f_{c,i}(\dot{q}_i)=\tanh(\dot{q}_i)$ for $i\in\{1,2\}$. Finally, τ_i is the input torque for joint i and τ_{ai} is given by (14).

For the simulation, the joint observer gains in (25) are chosen as $\ell_1 = 200$ so that $L_i = 210$ for both joints, and the link observer gains in (17) are chosen as $\mathbf{L}_{\mathbf{B}_i} = 200 \times I_{3\times 3}$ for both links. The link control gains in (33) are chosen as $\mathbf{K}_{\mathbf{B}_i} = 100 \times I_{3\times 3}$ for both links, and the joint control gains in (38) are chosen as $k_i = 10$ for both joints. Moreover, for the required velocities in (29) the control parameter is chosen as $\lambda_i = 10$ for both required velocities. The simulations are run on a Simulink model corresponding to the dynamics presented in the beginning of this section.

The desired joint trajectories are given by $q_{1d}(t) = 0.8 - \cos\left(\frac{\pi}{4}t\right)$ and $q_{2d}(t) = 0.8 - \cos\left(\frac{\pi}{5}t\right)$. The desired trajectories and the joint position trajectories are displayed in Fig. 2, where an initial error can be seen as the joints are initially at $q_1(0) = q_2(0) = 0$ whereas the desired values are -0.2. However, the initial error diminishes quickly, and thereafter the joint trajectories follow the desired trajectories accurately.

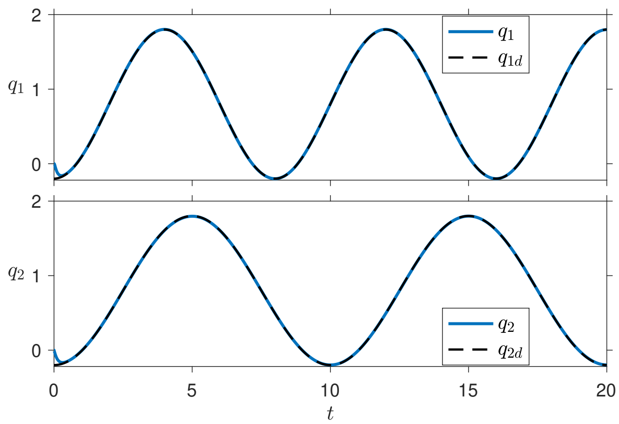

Fig. 2. Joint angle trajectories (in radians) and their desired values.

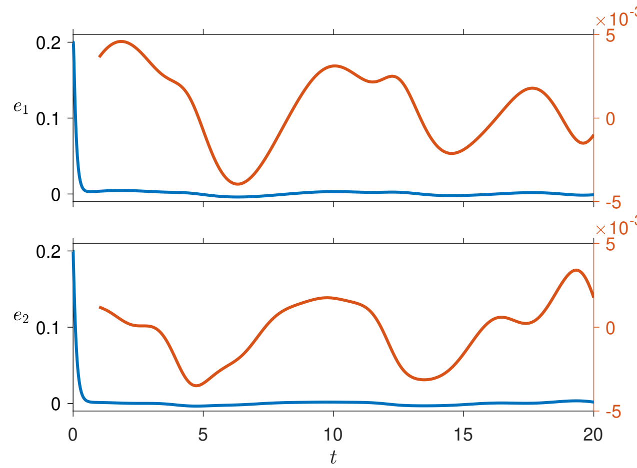

Fig. 3. Tracking errors $e_i = q_i - q_{id}$ (in radians) for joints 1 and 2. The asymptotic behavior for $t \geq 1$ is depicted in detail by the red lines and axes.

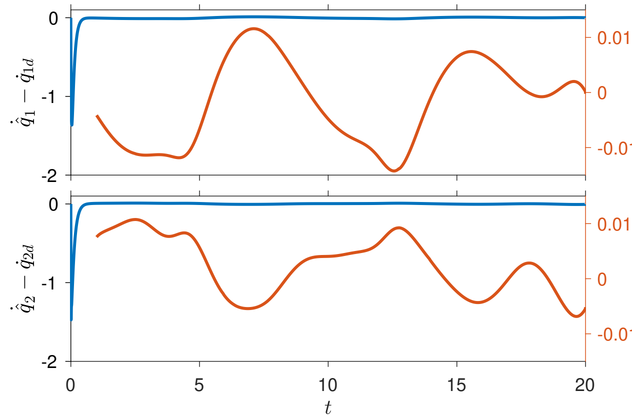

Fig. 4. Observer errors $\hat{\dot{q}}_i - \dot{q}_{id}$ (in radians/second) for joints 1 and 2. The asymptotic behavior for $t \geq 1$ is depicted in detail by the red lines and axes.

Fig. 3 displays the tracking errors $e_i = q_i - q_{id}$ for both joints 1 and 2. The tracking errors behave according to Fig. 2, that is, for both joints there is an initial error of 0.2 radians which diminishes rapidly, and thereafter the position errors are virtually zero. The velocity observer errors $\hat{\dot{q}}_i - \dot{q}_{id}$ are shown in Fig. 4, where one can see relatively large initial peaks as the initial position tracking error is adjusted by the control input, but thereafter the observed velocities are in accordance with the desired velocities.

VIII. Conclusions

We incorporated a decentralized velocity observer design in the framework of virtual decomposition control of an open chain robotic manipulator. Stability analysis for the proposed controller-observer was carried out on a subsystem level by utilizing the concept of virtual stability. The observer error dynamics for a single subsystem were found to be independent of the other subsystems, which would suggest that the design could be extended to more complex systems as noted in Remark VI.5. In addition to proving the semiglobal exponential convergence of the combined controller-observer design, the proposed design was demonstrated in a simulation study of a 2-DoF open chain system in the vertical plane. A topic for future research will be to incorporate parameter adaptation into the controller-observer design.

Appendix A
Proof for Lemma V.1

First note that by subtracting (11) from (32), we obtain

$$\begin{aligned} {}^{\mathbf{B}_i}F_r^* - {}^{\mathbf{B}_i}F^* = \mathbf{M}_{\mathbf{B}_i}\frac{d}{dt}({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V) + \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{\omega}){}^{\mathbf{B}_i}V_r \\ - \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\omega){}^{\mathbf{B}_i}V + \mathbf{K}_{\mathbf{B}_i}({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}\hat{V}). \end{aligned} \tag{49}$$

Utilizing the properties of $\mathbf{C}_{\mathbf{B}_i}(\cdot)$ as in Section IV and using the fact that $2V_1^T V_2 \leq \|V_1\|^2 + \|V_2\|^2$, we obtain

$$\begin{aligned} &({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)^T[\mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{\omega}){}^{\mathbf{B}_i}V_r - \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\omega){}^{\mathbf{B}_i}V] \\ &= ({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)^T\mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{\omega} - {}^{\mathbf{B}_i}\omega){}^{\mathbf{B}_i}V \\ &\leq \frac{1}{2}\|{}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V\|^2 + \frac{1}{2}\|\mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{\omega} - {}^{\mathbf{B}_i}\omega){}^{\mathbf{B}_i}V\|^2 \\ &\leq \frac{1}{2}\|{}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V\|^2 + \frac{1}{2}M_{c,i}^2\|{}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V\|^2 M_{v,i}^2 \end{aligned} \tag{50}$$

where we also used the boundedness assumption of ${}^{\mathbf{B}_i}V$ and the relative boundedness (5c) of $\mathbf{C}_{\mathbf{B}_i}(\cdot)$. Similarly, we obtain

$$\begin{aligned} &-({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)^T\mathbf{K}_{\mathbf{B}_i}({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}\hat{V}) \\ &\leq -({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)^T\mathbf{K}_{\mathbf{B}_i}({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V) \\ &\quad + \frac{1}{2}\|\sqrt{\mathbf{K}_{\mathbf{B}_i}}({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)\|^2 + \frac{1}{2}\|\sqrt{\mathbf{K}_{\mathbf{B}_i}}({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V)\|^2. \end{aligned} \tag{51}$$

Moreover, using (9), (12), (31) and (33) we obtain

$$\begin{aligned}
&({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)^T({}^{\mathbf{B}_i}F_r^* - {}^{\mathbf{B}_i}F^*) \\
&= ({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)^T\left[({}^{\mathbf{B}_i}F_r - {}^{\mathbf{B}_i}F) - {}^{\mathbf{B}_i}\mathbf{U}_{\mathbf{T}_i}({}^{\mathbf{T}_i}F_r - {}^{\mathbf{T}_i}F)\right] \\
&= p_{\mathbf{B}_i} - \left[{}^{\mathbf{B}_i}\mathbf{U}_{\mathbf{T}_i}^T({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)\right]^T({}^{\mathbf{T}_i}F_r - {}^{\mathbf{T}_i}F) \\
&= p_{\mathbf{B}_i} - ({}^{\mathbf{T}_i}V_r - {}^{\mathbf{T}_i}V)^T({}^{\mathbf{T}_i}F_r - {}^{\mathbf{T}_i}F) = p_{\mathbf{B}_i} - p_{\mathbf{T}_i}.
\end{aligned} \tag{52}$$

Using (49)–(52) together with (24), $\nu_{\mathbf{B}_i}$ in (35) satisfies

$$\begin{aligned}
\dot{\nu}_{\mathbf{B}_i} &= \dot{\nu}_{\mathbf{B}_i,ctrl} + \dot{\nu}_{\mathbf{B}_{i,obs}} \\
&= -({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)^T[\mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\hat{\omega}){}^{\mathbf{B}_i}V_r - \mathbf{C}_{\mathbf{B}_i}({}^{\mathbf{B}_i}\omega){}^{\mathbf{B}_i}V] \\
&\quad - ({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)^T\mathbf{K}_{\mathbf{B}_i}({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}\hat{V}) \\
&\quad + ({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)^T({}^{\mathbf{B}_i}F_r^* - {}^{\mathbf{B}_i}F^*) + \dot{\nu}_{\mathbf{B}_{i,obs}} \\
&\le \frac{1}{2}\|{}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V\|^2 + \frac{1}{2}M_{c,i}^2\|{}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V\|^2 M_{v,i}^2 \\
&\quad - ({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)^T\mathbf{K}_{\mathbf{B}_i}({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V) + p_{\mathbf{B}_i} - p_{\mathbf{T}_i} \\
&\qquad + \frac{1}{2}\|\sqrt{\mathbf{K}_{\mathbf{B}_i}}({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)\|^2 + \frac{1}{2}\|\sqrt{\mathbf{K}_{\mathbf{B}_i}}({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V)\|^2 \\
&\quad - ({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V)^T(\mathbf{L}_{\mathbf{B}_i} - M_{c,i}M_{v,i}I_{6\times6})({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V) \\
&= -({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V)^T\left(\frac{1}{2}\mathbf{K}_{\mathbf{B}_i} - \frac{1}{2}I_{6\times6}\right)({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V) \\
&\quad - ({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V)^T \\
&\qquad \times\left[\mathbf{L}_{\mathbf{B}_i} - M_{c,i}M_{v,i}\left(1 + \frac{1}{2}M_{c,i}M_{v,i}\right)I_{6\times6} - \frac{1}{2}\mathbf{K}_{\mathbf{B}_i}\right] \\
&\qquad \times({}^{\mathbf{B}_i}\hat{V} - {}^{\mathbf{B}_i}V) + p_{\mathbf{B}_i} - p_{\mathbf{T}_i},
\end{aligned}$$

and the claim follows.

Appendix B
Proof for Lemma V.2

First note that by subtracting (15) from (38b), we obtain

$$\begin{aligned}
\tau_{air} - \tau_{ai} = &- I_{m,i}(\ddot{q}_{ir} - \ddot{q}_i) - [f_{c,i}(\dot{q}_{ir}) - f_{c,i}(\dot{q}_i)] \\
&- k_{q,i}(\dot{q}_{ir} - \dot{\hat{q}}_i).
\end{aligned} \tag{53}$$

Using (6), (8), (14), (13), (30), and (38) we obtain for $i = 1$ that

$$\begin{aligned}
&(\dot{q}_{1r} - \dot{q}_1)(\tau_{a1r} - \tau_{a1}) \\
&= (\dot{q}_{1r} - \dot{q}_1)\mathbf{z}_\tau^T({}^{\mathbf{B}_1}F_r - {}^{\mathbf{B}_1}F) \\
&= [{}^{\mathbf{B}_1}V_r - {}^{\mathbf{B}_1}V - {}^{\mathbf{B}_0}U_{\mathbf{B}_1}^T({}^{\mathbf{B}_0}V_r - {}^{\mathbf{B}_0}V)]^T({}^{\mathbf{B}_1}F_r - {}^{\mathbf{B}_1}F) \\
&= p_{\mathbf{B}_1} - ({}^{\mathbf{B}_0}V_r - {}^{\mathbf{B}_0}V)^T{}^{\mathbf{B}_0}U_{\mathbf{B}_1}({}^{\mathbf{B}_1}F_r - {}^{\mathbf{B}_1}F) \\
&= p_{\mathbf{B}_1} - ({}^{\mathbf{B}_0}V_r - {}^{\mathbf{B}_0}V)^T({}^{\mathbf{B}_0}F_r - {}^{\mathbf{B}_0}F) = p_{\mathbf{B}_1} - p_{\mathbf{B}_0}.
\end{aligned} \tag{54}$$

Similarly, using (6), (10), (14), (37) and (38), we obtain for $i \in \{2, 3, \ldots, n\}$ that

$$\begin{aligned}
&(\dot{q}_{ir} - \dot{q}_i)(\tau_{air} - \tau_{ai}) \\
&= (\dot{q}_{ir} - \dot{q}_i)\mathbf{z}_\tau^T({}^{\mathbf{B}_i}F_r - {}^{\mathbf{B}_i}F) \\
&= \left[({}^{\mathbf{B}_i}V_r - {}^{\mathbf{B}_i}V) - {}^{\mathbf{T}_{i-1}}\mathbf{U}_{\mathbf{B}_i}^T({}^{\mathbf{T}_{i-1}}V_r - {}^{\mathbf{T}_{i-1}}V)\right]^T({}^{\mathbf{B}_i}F_r - {}^{\mathbf{B}_i}F) \\
&= p_{\mathbf{B}_i} - ({}^{\mathbf{T}_{i-1}}V_r - {}^{\mathbf{T}_{i-1}}V)^T{}^{\mathbf{T}_{i-1}}\mathbf{U}_{\mathbf{B}_i}({}^{\mathbf{B}_i}F_r - {}^{\mathbf{B}_i}F) \\
&= p_{\mathbf{B}_i} - ({}^{\mathbf{T}_{i-1}}V_r - {}^{\mathbf{T}_{i-1}}V)^T({}^{\mathbf{T}_{i-1}}F_r - {}^{\mathbf{T}_{i-1}}F) = p_{\mathbf{B}_i} - p_{\mathbf{T}_{i-1}}.
\end{aligned} \tag{55}$$

Using (53)–(55) together with (28), ν_{ai} in (39) satisfies

$$\begin{aligned}
\dot{\nu}_{ai} = &-k_i(\dot{q}_{ir} - \dot{q}_i)^2 - [f_{c,i}(\dot{q}_{ir}) - f_{c,i}(\dot{q}_i)](\dot{q}_{ir} - \dot{q}_i) \\
&- (\dot{q}_{ir} - \dot{q}_i)(\tau_{air} - \tau_{ai}) + k_i(\dot{q}_{ir} - \dot{q}_i)(\dot{\hat{q}}_i - \dot{q}_i) + \dot{\nu}_{i,obs} \\
\le &-\frac{1}{2}k_i(\dot{q}_{ir} - \dot{q}_i)^2 - \left(I_{m,i}L_i - \frac{m_{c,i}^2 + k_i}{2}\right)(\dot{\hat{q}}_i - \dot{q}_i)^2 \\
&- \frac{1}{2}s_i^2 + p_{\mathbf{T}_{i-1}} - p_{\mathbf{B}_i},
\end{aligned}$$

and the claim follows.

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