# *Backstepping Techniques for PDE Control: From Fundamentals to Open Problems*

Federico Bribiesca-Argomedo

INSA Lyon, Laboratoire Ampère (UMR 5005)
Villeurbanne, France

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*Backstepping for partial differential equations: A survey*
R. Vazquez, J. Auriol, F. Bribiesca-Argomedo, M. Krstic

# Why control systems modeled by PDEs?

PDEs model systems that vary in *space and time*

- traffic flow, heat & chemical reactors, flexible structures

- batteries, drilling, plasmas, fluid flows

We often want to **stabilize**, **estimate**, or **improve performance**

Yet ready-to-use, systematic design tools have historically been scarce

*Backstepping* offers a **constructive**, often **explicit** answer for boundary control

# What this talk is (and is not)

Goal: build intuition for the backstepping method, survey where the field is going, and open today's program

**Three parts:**

- **Fundamentals**: the method on one paradigmatic example

- **Adaptations & applications**: what the basic recipe needs in practice

- **Open problems**: where a starting researcher can contribute

Boundary control of 1-D linear PDEs is the backbone; pointers to the rest.

This is **not** a complete overview (time constraints) nor can we cite hundreds of papers (go read the survey!)

# Part I: Fundamentals

The method in one picture, on one example

# Backstepping in one picture

**Main idea:** do not tackle the original (unstable/difficult-to-solve) PDE directly. *Change coordinates* so that it becomes a system you already know is stable

$$\begin{tikzcd} u(\cdot,0) \arrow[r, "T"] \arrow[d, "e^{\mathcal{A}_{CL}t}"'] & w(\cdot,0) \arrow[d, "e^{\mathcal{A}_d t}"] \\ u(\cdot,t) & w(\cdot,t) \arrow[l, "T^{-1}"] \end{tikzcd}$$

plant $\xrightarrow{T}$ stable target system

If $T$ is **bounded** and **boundedly invertible**, the two share stability properties (Smyshlyaev and Krstic, 2004)

# A paradigmatic plant: reaction-diffusion

Consider a reaction-diffusion PDE on $x \in [0, 1]$:

$$u_t = \epsilon u_{xx} + \lambda(x)u, \quad u(t, 0) = 0, \quad u(t, 1) = U(t)$$

- $\epsilon > 0$: diffusion $\lambda(x)$: reaction (may be *destabilizing*)

- $U(t)$: boundary actuation (the only place where we can act!)

**Open loop** ($U = 0$): a Lyapunov estimate with $L(t) = \frac{1}{2}\|u\|_{L^2}^2$ gives

$$\|u(t)\|_{L^2}^2 \leq \mathrm{e}^{-(\frac{\epsilon}{2} - 2\bar{\lambda})t}\|u(0)\|_{L^2}^2$$

exponentially stable if $\bar{\lambda} := \max_x \lambda < \epsilon/4$ (not sharp); a large reaction can destabilize, and then we must act

# The three interlinked ingredients

A backstepping design rests on **three** key elements (Smyshlyaev and Krstic, 2004):

1. a target system with the properties we want (e.g., stability)
2. a transformation mapping plant → target, bounded & invertible
3. kernel equations: the conditions the transform must satisfy

They are **coupled**: a good target ⇒ solvable kernels + invertible transform. A bad target ⇒ no solution or not invertible.

# Step 1: choosing the target system

For our plant, a natural target neutralizes the reaction term and adds a degree of freedom:

$$w_t = \epsilon w_{xx} - c\, w, \quad w(t,0) = 0, \quad w(t,1) = 0$$

exponentially stable for any $c \geq 0$ (decay rate tunable via $c$)

## Design principles

- *keep the nature of the system (still reaction-diffusion, same $\epsilon$)*
- *keep the boundary conditions (Dirichlet stays Dirichlet)*
- *transform only the destabilizing terms (cancel the reaction or make it stabilizing)*

Similar enough to be reachable, different enough to be stable.

# Step 2: the backstepping transformation

ODE backstepping (Krstic, Kanellakopoulos, and Kokotovic, 1995) applied to spatial discretizations of the PDE (Balogh and Krstic; Bošković, Balogh, and Krstić, 2002; 2003) suggests a *spatially causal* (strict-feedback) map:

$$w(t,x) = u(t,x) - \int_0^x k(x,\xi)\, u(t,\xi)\, d\xi$$

- a **Volterra** integral (upper limit $x$): the value at $x$ depends only on $[0,x]$

- Volterra $\Rightarrow$ **always invertible** under mild conditions on $k$

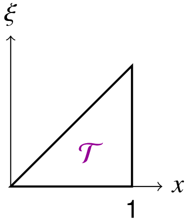
When strict-feedback structure is lost, one needs a **Fredholm** map, and invertibility is no longer free (Part III)

# Step 3: the kernel equations

Substitute the transform into the target, integrate by parts: $k$ must solve a hyperbolic PDE on the triangle $\mathcal{T} = \{0 \leq \xi \leq x \leq 1\}$:

$$\epsilon k_{xx} - \epsilon k_{\xi\xi} = (\lambda(\xi) + c)\, k,$$

$$k(x,0) = 0, \qquad k(x,x) = -\frac{1}{2\epsilon}\int_0^x (\lambda(\xi) + c)\, d\xi$$



# Well-posedness: successive approximations

Change variables, integrate once $\Rightarrow$ an *integral equation* $G = G_0 + \mathcal{K}[G]$

Solve by iterating:

$$k = \lim_{n\to\infty} G_n, \qquad G_n = G_0 + \mathcal{K}[G_{n-1}]$$

Since $\lambda$ is continuous, the series **converges**: a unique $k \in C^1(\mathcal{T})$

This same recipe (*integral form* + *successive approximations*) recurs for almost every backstepping design in the talk (Fredholm transforms require a different approach)

# The control law (and what it needs)

Evaluate the transform at the actuated boundary $x = 1$, use $w(t, 1) = 0$:

$$U(t) = \int_0^1 k(1, \xi)\, u(t, \xi)\, d\xi$$

Observations that matter in practice:

- the *implementation* uses only the **trace** $k(1, \cdot)$; the full kernels $k$, $l$ still enter the stability and robustness estimates (next slide)

- evaluating a trace demands **regularity**: a bounded $k$ is not enough, in this example, the continuity from $k \in C^1(\mathcal{T})$ makes the gain well-defined

- it needs the **full state** $u(t, \cdot)$ $\Rightarrow$ motivates observers

# Why the closed loop is stable

The Volterra transform is invertible: $u = w + \int_0^x l(x,\xi) w \, d\xi$

Direct + inverse give two-sided norm equivalence:

$$||w||_{L^2} \leq (1 + ||k||_\infty)||u||_{L^2}, \quad ||u||_{L^2} \leq (1 + ||l||_\infty)||w||_{L^2}$$

**Theorem**

*The origin of the closed-loop $u$-system is exponentially stable in $L^2$:*

$$||u(t)||_{L^2}^2 \leq (1 + ||l||_\infty)^2 (1 + ||k||_\infty)^2 \, \mathrm{e}^{-(\frac{\epsilon}{2}+2c)t} ||u(0)||_{L^2}^2$$

Decay rate is *chosen*; constant in the estimate grows with the kernel size (a recurring trade-off).

# Observers by duality

We rarely measure the whole state. Suppose we only measure $u_x(t,0)$, at the *unactuated* end (anti-collocated).

Observer = copy of the plant + output-error injection $p_1(x)$:

$$\hat{u}_t = \epsilon \hat{u}_{xx} + \lambda \hat{u} + p_1(x)\big(u_x(t,0) - \hat{u}_x(t,0)\big)$$

The error $\tilde{u} = u - \hat{u}$ is mapped to a stable target by a *dual* backstepping transform; its kernel equations are the *same type* as before (Smyshlyaev and Krstic, 2005)

$\Rightarrow$ the gain $p_1$ is obtained directly from the observer kernel. Design once, reuse by duality.

# Output feedback + other 1-D systems

Combine controller (needs state) with observer (provides estimate):

$$U(t) = \int_0^1 k(1,\xi)\,\hat{u}(t,\xi)\,d\xi$$

For linear PDEs the **separation principle** holds: controller + observer is stable.

The *same* three-step recipe extends to:

- first-order hyperbolic (transport) with some non-local terms: target is a pure transport (delay), finite-time stable (Krstic and Smyshlyaev, 2008a)

- wave / beams: introduce damping in the target (Krstic, Siranosian, and Smyshlyaev, 2006)

- complex-valued (Schrödinger): kernels via Bessel/Kelvin functions (Krstic, Guo, and Smyshlyaev, 2011)

# Part II: Adaptations & applications

What the basic recipe needs in practice

# Interconnected systems: the general picture

Real systems may couple transport, in-domain effects, and lumped (ODE) dynamics:

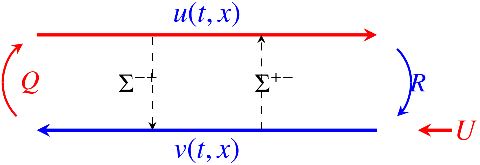
$$\begin{cases} \dot{X} = A_0 X + BU(t) + E_0 v(t,0) \quad \text{(actuator ODE)} \\ w_t + \Lambda(x) w_x = \Sigma(x) w, \quad w = (u^\top, v^\top)^\top \\ \dot{Y} = A_1 Y + E_1 u(t,1) \quad \text{(load ODE)} \\ u(0) = Qv(0) + C_0 X, \quad v(1) = Ru(1) + C_1 Y \end{cases}$$

$u/v$: right/left-going states $\Lambda$: speeds $\Sigma$: in-domain coupling $Q, R$: boundary reflections

The transform gains block-triangular structure (PDE part + ODE terms), still boundedly invertible

# From $2 \times 2$ to $n + m$ heterodirectional

$n$ rightward + $m$ leftward transport states, coupled in-domain and at boundaries.




- controllable to zero in minimum time $t_F = \frac{1}{\lambda_1} + \frac{1}{\mu_1}$ (slowest speed each way) (Li and Rao; Coron, Vazquez, Krstic, and Bastin, 2010; 2013)

- $n + m$ case: controllers & observers for the general case (Hu, Di Meglio, Vazquez, and Krstic; Auriol and Di Meglio; Coron, Hu, and Olive, 2016; 2016; 2017)

# A crucial caveat: delay-robustness

Method of characteristics rewrites the hyperbolic system as an **Integral Delay Equation** for the transformed state $Z$ at the actuated boundary (Auriol and Di Meglio, 2019):

$$Z(t) = \sum_{i=1}^{n_0} F_i\, Z(t-\tau_i) + \int_0^{t_F} H(\nu) Z(t-\nu)\, d\nu + U$$

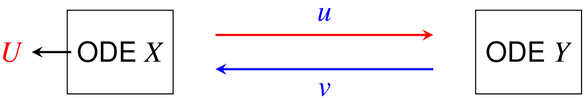
Cancelling *all* reflections gives simple finite-time proofs, but can leave a **zero delay margin** (Auriol, Aarsnes, Martin, and Di Meglio, 2018a)

For robustness: open loop (no $\Sigma$, no ODE) already stable; for $n = m = 1$ this is simply $|\rho q| < 1$ (scalar boundary reflections)

$\Rightarrow$ do not over-actuate: keep some reflection, do not demand infinite bandwidth from the actuators

# PDE-ODE and ODE-PDE-ODE interconnections

Actuator and/or load dynamics at the boundaries (drilling, delays, flexible structures)




- naive designs cancel everything $\Rightarrow$ robustness lost (Di Meglio, Bribiesca-Argomedo, Hu, and Krstic; Bou Saba, Bribiesca-Argomedo, Di Loreto, and Eberard, 2018; 2017)

- robust route: preserve some reflection, rewrite as neutral/IDE, use **filtering** (Auriol, Bribiesca-Argomedo, Bou Saba, Di Loreto, and Di Meglio; Di Meglio, Lamare, and Aarsnes; Auriol, Bribiesca-Argomedo, and Di Meglio, 2018b; 2020; 2023a)

- a **strictly proper** controller (low-pass filter) limits bandwidth (Bou Saba, Bribiesca-Argomedo, Di Loreto, and Eberard, 2019)

# More adaptations, at a glance

The same philosophy, retuned for structure:

- **coupled parabolic** $n \times n$: matrix kernels; kernel PDEs mirror the hyperbolic ones (Vazquez and Krstic, 2017)

- **time-varying** coefficients: Gevrey regularity (parabolic) (Vazquez, Trélat, and Coron; Kerschbaum and Deutscher, 2008; 2019) vs. easier hyperbolic (Coron and Nguyen, 2021); gain-scheduling (Auriol and Espitia, 2024)

- **bilateral** control: integrate from the middle of the domain (Vazquez and Krstic, 2016a)

- **beams** (wave & Timoshenko): Riemann transform → hyperbolic PDE-ODE (Su, Guo, Wang, and Krstic; Chen, Vazquez, and Krstic, 2017; 2023)

- **higher dimensions**: disks and spheres via symmetry (Vazquez and Krstic; Vazquez and Krstic, 2016b; 2019)

- **adaptive** designs for unknown parameters (Anfinsen and Aamo; Smyshlyaev and Krstic, 2019; 2010)

# Applications gallery

Backstepping has shown its versatility across many application areas:

- **Traffic**: stop-and-go on freeways (ARZ model), ramp metering (Yu and Krstic; Yu, Auriol, and Krstic, 2019; 2022)
- **Flow control**: Navier-Stokes / MHD channel flows (Vazquez and Krstic; Xu, Schuster, Vazquez, and Krstic, 2007; 2008)
- **Drilling**: axial/torsional vibrations, ODE-PDE-ODE (Aarsnes and van de Wouw; Bresch-Pietri and Krstic, 2019; 2014)
- **Energy**: Li-ion batteries, Stefan (melting) problems (Moura, Chaturvedi, and Krstic; Koga, Diagne, and Krstic, 2013; 2018)
- **Thermo-acoustics**: the Rijke tube (de Andrade, Vazquez, and Pagano, 2018)
- **Multi-agent**: continuum deployment / formation (Frihauf and Krstic, 2010)

Mostly theory/simulation; *experimental* validation so far includes the Rijke tube (de Andrade, Vazquez, and Pagano, 2020), multi-agent robots (Freudenthaler and Meurer, 2020), and a traffic observer on real freeway data (Yu, Gan, Bayen, and Krstic, 2021).

# Implementation: solving the kernels

The controller is implementable only after we compute $k(1, \cdot)$:

- **explicit** solutions in special cases (Bessel functions) (Krstic and Smyshlyaev, 2008b)

- **numerics**: finite differences, integral form, or **power series** (Vazquez, Chen, Qiao, and Krstic, 2023a)

- **late lumping**: approximate the controller last; needs stability guarantees

- **neural operators**: learn kernels or gains, with convergence guarantees (Bhan, Shi, and Krstic; Vazquez and Krstic, 2024; 2024)

# Part III: Open problems

Where a starting PhD can contribute

# Underactuated systems

Almost all designs assume a **fully actuated boundary**.

Underactuated: fewer inputs than boundary states,
$v(t,1) = Ru(t,1) + BU(t)$ with $\operatorname{rank}(B) < \dim v$

- the controllability guarantees behind full actuation (Li and Rao, 2010) no longer apply

- one attempt for a $1+2$ system with a single input (Auriol, Bribiesca-Argomedo, and Bresch-Pietri, 2020b)

- other results need structural assumptions (cascades) (Auriol; Deutscher, Gehring, and Jung, 2024; 2024)

Largely **open**, and central to networks of PDEs.

# In-domain actuation

The Volterra structure made backstepping a **boundary** method by design.

Actuation/sensing *inside* the domain has resisted generic results:

- early parabolic results need special actuator shapes / domain lengths

- hyperbolic results link to **Fredholm** transforms (Redaud and Auriol; Zhang, 2023; 2019)

Progress on Fredholm backstepping (Coron, Hu, and Olive, 2016) may unlock this (some results by the community might be in the works, no spoilers here).

# Parametrizable target systems

We usually choose the *simplest* target (cancel everything), but that costs robustness.

What if the target had **tuning parameters**? For instace, for coupled transport equations:

$$\begin{cases} \gamma_t + \Lambda(x)\gamma_x = \bar{\Sigma}(x)\gamma \\ \alpha(0) = Q\beta(0), \quad \beta(1) = \bar{R}\,\alpha(1) \end{cases}$$

- such targets are always reachable (Redaud, Auriol, and Le Gorrec, 2024); Port-Hamiltonian ideas guide the choice

- trade off delay-robustness against convergence rate (Auriol, Aarsnes, Di Meglio, and Shor, 2020a); degrees of freedom make the design **modular**

*Quantifying* the effect of these parameters is open.

# Stochastic systems & risk-averse design

Disturbances are usually random, yet most robust designs are *worst-case*.

Early results, in the mean-square sense:

- prediction-based control for ODEs with stochastic input delays (Kong and Bresch-Pietri, 2022)

- stability analysis of $2 \times 2$ hyperbolic systems with random parameters (Auriol, Pereira, and Kulcsar, 2023b)

Open: control the **variance**, not just the mean (**risk-averse** backstepping).

# Multi-dimensional theory & learning

Despite progress, backstepping remains fundamentally **1-D**.

Multi-D results rely on special geometries (balls, disks, spheres) + symmetry (Vazquez and Krstic; Vazquez and Krstic; Vazquez, Zhang, Qi, and Krstic, 2016b; 2019; 2023b)

A general multi-D theory would need:

- target systems in several dimensions

- well-posed multi-D kernel equations + practical solvers

Domain-extension for irregular domains is one promising route (Vazquez, 2025); **neural operators** are a complementary one: learn kernels, keep guarantees.

# Conclusions

Backstepping = **target system** + **invertible transform** + **kernel equations**

- constructive, often explicit, with precise closed-loop knowledge
- the same procedure extends from a scalar heat equation to PDE-ODE networks
- key practical lesson: *do not over-actuate* (robustness needs restraint)
- fertile open ground: underactuation, in-domain, stochastic, multi-D, learning

Today's program continues these threads (bridging theory and implementation):

- interconnected PDE-ODE backstepping, systematically: **N. Gehring** (next)
- Port-Hamiltonian target systems: **H. Ramirez**, **A. Macchelli**
- neural-operator and data-driven PDE control: **Y. Shi**, **G. A. de Andrade**

*Thank you! Questions welcome.*

# References I

U. J. F. Aarsnes and N. van de Wouw. Axial and torsional self-excited vibrations of a distributed drill-string. *Journal of Sound and Vibration*, 444:127–151, 2019.

H. Anfinsen and O. M. Aamo. *Adaptive control of hyperbolic PDEs*. Communications and Control Engineering. Springer, 2019.

J. Auriol. *Contributions to the robust stabilization of networks of hyperbolic systems*. HDR, Université Paris Saclay, 2024.

J. Auriol and F. Di Meglio. Minimum time control of heterodirectional linear coupled hyperbolic PDEs. *Automatica*, 71:300–307, 2016.

J. Auriol and F. Di Meglio. An explicit mapping from linear first order hyperbolic PDEs to difference systems. *Systems & Control Letters*, 123:144–150, 2019.

J. Auriol and N. Espitia. Event-triggered gain scheduling of $2\times 2$ hyperbolic PDEs with time and space varying coupling coefficients. In *IEEE Conference on Decision and Control*, 2024.

J. Auriol, U. J. F. Aarsnes, P. Martin, and F. Di Meglio. Delay-robust control design for heterodirectional linear coupled hyperbolic PDEs. *IEEE Transactions on Automatic Control*, 63(10):3551–3557, 2018a.

# References II


J. Auriol, F. Bribiesca-Argomedo, D. Bou Saba, M. Di Loreto, and F. Di Meglio. Delay-robust stabilization of a hyperbolic PDE–ODE system. *Automatica*, 95:494–502, 2018b.

J. Auriol, U. J. F. Aarsnes, F. Di Meglio, and R. Shor. Robust control design of underactuated 2×2 PDE-ODE-PDE systems. *IEEE Control Systems Letters*, 5(2): 469–474, 2020a.

J. Auriol, F. Bribiesca-Argomedo, and D. Bresch-Pietri. Stabilization of an underactuated 1+2 linear hyperbolic system with a proper control. In *2020 59th IEEE Conference on Decision and Control (CDC)*, 2020b.

J. Auriol, F. Bribiesca-Argomedo, and F. Di Meglio. Robustification of stabilizing controllers for ODE-PDE-ODE systems: a filtering approach. *Automatica*, 147:110724, 2023a.

J. Auriol, M. Pereira, and B. Kulcsar. Mean-square exponential stabilization of coupled hyperbolic systems with random parameters. In *IFAC World Congress 2023*, 2023b.

A. Balogh and M. Krstic. Infinite dimensional backstepping-style feedback transformations for a heat equation with an arbitrary level of instability. *European Journal of Control*, 8 (2):165–175, 2002.

# References III

L. Bhan, Y. Shi, and M. Krstic. Neural operators for bypassing gain and control computations in PDE backstepping. *IEEE Transactions on Automatic Control*, 69(8): 5310–5325, 2024. doi: 10.1109/TAC.2023.3347499.

D. M. Bošković, A. Balogh, and M. Krstić. Backstepping in infinite dimension for a class of parabolic distributed parameter systems. *Mathematics of Control, Signals, and Systems*, 16(1):44–75, 2003.

D. Bou Saba, F. Bribiesca-Argomedo, M. Di Loreto, and D. Eberard. Backstepping stabilization of 2×2 linear hyperbolic PDEs coupled with potentially unstable actuator and load dynamics. In *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*, pages 2498–2503, 2017. doi: 10.1109/CDC.2017.8264016.

D. Bou Saba, F. Bribiesca-Argomedo, M. Di Loreto, and D. Eberard. Strictly proper control design for the stabilization of $2 \times 2$ linear hyperbolic ODE-PDE-ODE systems. In *2019 IEEE 58th Conference on Decision and Control (CDC)*, pages 4996–5001. IEEE, 2019.

D. Bresch-Pietri and M. Krstic. Adaptive output feedback for oil drilling stick-slip instability modeled by wave PDE with anti-damped dynamic boundary. In *American Control Conference (ACC), 2014*, pages 386–391. IEEE, 2014.

# References IV

G. Chen, R. Vazquez, and M. Krstic. Rapid stabilization of Timoshenko beam by PDE backstepping. *IEEE Transactions on Automatic Control*, 69(2):1141–1148, 2023.

J.-M. Coron and H.-M. Nguyen. On the optimal controllability time for linear hyperbolic systems with time-dependent coefficients. *arXiv preprint arXiv:2103.02653*, 2021.

J.-M. Coron, R. Vazquez, M. Krstic, and G. Bastin. Local exponential $H^2$ stabilization of a $2\times$ 2 quasilinear hyperbolic system using backstepping. *SIAM Journal on Control and Optimization*, 51(3):2005–2035, 2013.

J.-M. Coron, L. Hu, and G. Olive. Stabilization and controllability of first-order integro-differential hyperbolic equations. *Journal of Functional Analysis*, 271(12): 3554–3587, 2016.

J.-M. Coron, L. Hu, and G. Olive. Finite-time boundary stabilization of general linear hyperbolic balance laws via Fredholm backstepping transformation. *Automatica*, 84: 95–100, 2017.

G. A. de Andrade, R. Vazquez, and D. J. Pagano. Backstepping stabilization of a linearized ODE–PDE Rijke tube model. *Automatica*, 96:98–109, 2018.

# References V


G. A. de Andrade, R. Vazquez, and D. J. Pagano. Backstepping-based estimation of thermoacoustic oscillations in a Rijke tube with experimental validation. *IEEE Transactions on Automatic Control*, 65(12):5336–5343, 2020. doi: 10.1109/TAC.2020.2970152.

J. Deutscher, N. Gehring, and N. Jung. Backstepping control of coupled general hyperbolic-parabolic PDE-PDE systems. *IEEE Transactions on Automatic Control*, 69 (5):3465–3472, 2024. doi: 10.1109/TAC.2023.3340119.

F. Di Meglio, F. Bribiesca-Argomedo, L. Hu, and M. Krstic. Stabilization of coupled linear heterodirectional hyperbolic PDE–ODE systems. *Automatica*, 87:281–289, 2018.

F. Di Meglio, P.-O. Lamare, and U. J. F. Aarsnes. Robust output feedback stabilization of an ODE–PDE–ODE interconnection. *Automatica*, 119:109059, 2020.

G. Freudenthaler and T. Meurer. PDE-based multi-agent formation control using flatness and backstepping: Analysis, design and robot experiments. *Automatica*, 115:108897, 2020. ISSN 0005-1098.

P. Frihauf and M. Krstic. Leader-enabled deployment onto planar curves: A PDE-based approach. *IEEE Transactions on Automatic Control*, 56(8):1791–1806, 2010.

# References VI


L. Hu, F. Di Meglio, R. Vazquez, and M. Krstic. Control of homodirectional and general heterodirectional linear coupled hyperbolic PDEs. *IEEE Transactions on Automatic Control*, 61(11):3301–3314, 2016.

S. Kerschbaum and J. Deutscher. Backstepping control of coupled linear parabolic PDEs with space and time dependent coefficients. *IEEE Transactions on Automatic Control*, 65(7):3060–3067, 2019.

S. Koga, M. Diagne, and M. Krstic. Control and state estimation of the one-phase Stefan problem via backstepping design. *IEEE Transactions on Automatic Control*, 64(2): 510–525, 2018.

S. Kong and D. Bresch-Pietri. Prediction-based controller for linear systems with stochastic input delay. *Automatica*, 138:110149, 2022.

M. Krstic and A. Smyshlyaev. Backstepping boundary control for first-order hyperbolic PDEs and application to systems with actuator and sensor delays. *Systems & Control Letters*, 57(9):750 – 758, 2008a.

M. Krstic and A. Smyshlyaev. *Boundary control of PDEs: A course on backstepping designs*, volume 16. SIAM, 2008b.

# References VII

M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic. *Nonlinear and Adaptive Control Design*. Wiley, New York, 1995.

M. Krstic, A. A. Siranosian, and A. Smyshlyaev. Backstepping boundary controllers and observers for the slender Timoshenko beam: Part I-Design. In *2006 American Control Conference*, pages 2412–2417. IEEE, 2006.

M. Krstic, B.-Z. Guo, and A. Smyshlyaev. Boundary controllers and observers for the linearized Schrödinger equation. *SIAM Journal on Control and Optimization*, 49(4): 1479–1497, 2011.

T. Li and B. Rao. Strong (weak) exact controllability and strong (weak) exact observability for quasilinear hyperbolic systems. *Chinese Annals of Mathematics, Series B*, 31(5): 723–742, 2010.

S. J. Moura, N. A. Chaturvedi, and M. Krstic. Adaptive partial differential equation observer for battery state-of-charge/state-of-health estimation via an electrochemical model. *Journal of Dynamic Systems, Measurement, and Control*, 136(1):011015, 2013. ISSN 0022-0434. doi: 10.1115/1.4024801. URL `https://doi.org/10.1115/1.4024801`.

# References VIII


J. Redaud and J. Auriol. Backstepping stabilization of a clamped string with actuation inside the domain. *IFAC-PapersOnLine*, 56(2):9936–9941, 2023. ISSN 2405-8963. doi: https://doi.org/10.1016/j.ifacol.2023.10.692. URL https://www.sciencedirect.com/science/article/pii/S2405896323010674. 22nd IFAC World Congress.

J. Redaud, J. Auriol, and Y. Le Gorrec. In domain dissipation assignment of boundary controlled Port-Hamiltonian systems using backstepping. *Systems & Control Letters*, 185:105722, 2024.

A. Smyshlyaev and M. Krstic. Closed-form boundary state feedbacks for a class of 1-D partial integro-differential equations. *IEEE Transactions on Automatic Control*, 49(12): 2185–2202, 2004.

A. Smyshlyaev and M. Krstic. Backstepping observers for a class of parabolic PDEs. *Systems & Control Letters*, 54(7):613–625, 2005.

A. Smyshlyaev and M. Krstic. *Adaptive Control of Parabolic PDEs*. Princeton University Press, 2010.

L. Su, W. Guo, J.-M. Wang, and M. Krstic. Boundary stabilization of wave equation with velocity recirculation. *IEEE Transactions on Automatic Control*, 62(9):4760–4767, 2017.

# References IX

R. Vazquez. Backstepping control laws for higher-dimensional PDEs: Spatial invariance and domain extension methods. *arXiv preprint arXiv:2503.00225*, 2025.

R. Vazquez and M. Krstic. A closed-form feedback controller for stabilization of the linearized 2-D Navier–Stokes Poiseuille system. *IEEE Transactions on Automatic Control*, 52(12):2298–2312, 2007.

R. Vazquez and M. Krstic. Bilateral boundary control of one-dimensional first-and second-order PDEs using infinite-dimensional backstepping. In *Conference on Decision and Control (CDC)*, pages 537–542. IEEE, 2016a.

R. Vazquez and M. Krstic. Boundary control of a singular reaction-diffusion equation on a disk. *IFAC-PapersOnLine*, 49(8):74–79, 2016b.

R. Vazquez and M. Krstic. Boundary control of coupled reaction-advection-diffusion systems with spatially-varying coefficients. *IEEE Transactions on Automatic Control*, 62: 2026–2033, 2017.

R. Vazquez and M. Krstic. Boundary control and estimation of reaction–diffusion equations on the sphere under revolution symmetry conditions. *International Journal of Control*, 92(1):2–11, 2019.

# References X


R. Vazquez and M. Krstic. Gain-only neural operators for PDE backstepping. *arXiv preprint arXiv:2403.19344*, 2024.

R. Vazquez, E. Trélat, and J.-M. Coron. Control for fast and stable laminar-to-high-Reynolds-numbers transfer in a 2D Navier-Stokes channel flow. *Discrete and Continuous Dynamical Systems - B*, 10(4):925–956, 2008. ISSN 1531-3492. doi: 10.3934/dcdsb.2008.10.925. URL `https://www.aimsciences.org/article/id/df441eaa-5ef9-47ca-809a-dbe7aeee04c7`.

R. Vazquez, G. Chen, J. Qiao, and M. Krstic. The power series method to compute backstepping kernel gains: theory and practice. In *2023 62nd IEEE Conference on Decision and Control (CDC)*, pages 8162–8169. IEEE, 2023a.

R. Vazquez, J. Zhang, J. Qi, and M. Krstic. Kernel well-posedness and computation by power series in backstepping output feedback for radially-dependent reaction–diffusion PDEs on multidimensional balls. *Systems & Control Letters*, 177:105538, 2023b.

C. Xu, E. Schuster, R. Vazquez, and M. Krstic. Stabilization of linearized 2D magnetohydrodynamic channel flow by backstepping boundary control. *Systems & Control Letters*, 57(10):805–812, 2008.

# References XI

H. Yu and M. Krstic. Traffic congestion control for Aw-Rascle-Zhang model. *Automatica*, 100:38–51, 2019.

H. Yu, Q. Gan, A. Bayen, and M. Krstic. PDE traffic observer validated on freeway data. *IEEE Transactions on Control Systems Technology*, 29(3), 2021.

H. Yu, J. Auriol, and M. Krstic. Simultaneous downstream and upstream output-feedback stabilization of cascaded freeway traffic. *Automatica*, 136:110044, 2022.

C. Zhang. Finite-time internal stabilization of a linear 1-D transport equation. *Systems & Control Letters*, 133:104529, 2019. ISSN 0167-6911. doi: https://doi.org/10.1016/j.sysconle.2019.104529. URL https://www.sciencedirect.com/science/article/pii/S0167691119301392.